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DISSERTATION

ROBUST ADAPTIVE CONTROL USING A FILTERING ACTION

by

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13. ABSTRACT (maximum 200 words)

This dissertation describes the design of an adaptive controller for single-input single-output (SISO) systems with guaranteed bounds on the transient response, and robustness with external disturbances and unmodeled dynamics. Developed from a current approach called “L1 adaptive controller”, we show that by adding two properly designed low pass filters at the input and at the estimator we can control the transient response and the sensitivity of the overall system to external disturbances and unmodeled dynamics. Global stability of the overall adaptive system is mathematically proven under the assumption that the system is minimum phase (i.e., with the zeros of the transfer function in the stable region) and bounds of the systems parameters are known to the designer.

The extension of this approach to non-minimum phase systems, such as systems with flexible appendages, is also considered. We show that a non-minimum phase plant augmented with a properly designed parallel system results in a minimum phase system. The augmenting system most easily comes from the inverse of a stabilizing Proportional-Integral-Derivative (PID) controller, designed to be least sensitive to parameter uncertainties. This approach is applied to a flexible arm in a testbed at the Naval Postgraduate School, called the Flexible Spacecraft Simulator (FSS), which emulates realistic conditions in space. Experimental results prove the effectiveness of the controller presented in this dissertation.

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ROBUST ADAPTIVE CONTROL USING A FILTERING ACTION

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LIST OF ACRONYMS AND ABBREVIATIONS

SISO	Single Input and Single Output
PID	Proportional Integral Derivative
FSS	Flexible Spacecraft Simulator
MRAC	Model Reference Adaptive Control
LPF	Low Pass Filter
LHS	Left Hand Side
MRC	Model Reference Control
I/O	Input-Output
LTI	Linear Time Invariant
NPS	Naval Postgraduate School
PPC	Pole Placement Control
SRDC	Satellite Research and Design Center
MIMO	Multi Input Multi Output

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EXECUTIVE SUMMARY

Adaptive control systems have been proposed to control systems with uncertain dynamics, widely varying under different operating conditions. They have been the subject of research for a number of years and stability has been established under ideal conditions. In spite of the amount of research performed on this class of control systems, sensitivity to external disturbances and modeling errors together with poor transient response performance made engineers skeptical to their applications. In particular, the Model Reference Adaptive Control (MRAC) approach, which seems to be the most effective under ideal conditions, can easily become unstable in the presence of unmodeled dynamics not accounted by the system model. Where instability means a complete loss of the platform, such as in space applications, adaptive control is far from being considered a possible solution to control problems.

In this dissertation, we address some of these issues, such as transient response and robustness in the presence of external disturbances and modeling errors. The proposed adaptive control system is derived from a recently introduced adaptive control approach called L1 Adaptive Control, with L1 referring to the norm of a certain operator affecting the tracking error. We show that the addition of two simple low pass filters, one at the control input and one at the parameter estimator, eliminates high frequency components, which adversely affect both the transient response and the sensitivity to disturbances. These two additional filters must be properly designed in order to guarantee stability and fast convergence.

A major drawback of this proposed scheme, relative to more traditional adaptive systems, is the need for a nominal model for the system and bounds on the parameter uncertainties. In other words, while transitional adaptive controllers assume the system is almost completely unknown (a black box), in the proposed scheme we assume the system to be within bounds (a gray box). In most, if not all applications, the gray box assumption is more realistic, since the order of magnitude of the dynamics is understood.

A further issue examined is the extension to non-minimum phase systems. This problem is addressed by replacing the actual system with an “augmented” system,

implemented by adding a dynamic system in parallel. It is easy to show that a non-proper stabilizing compensator, like a proportional-integral-derivative (PID) controller, can be used by defining the inverse of its transfer function (thus proper) as the augmenting system. The combination then becomes minimum phase, and it can be controlled by an adaptive controller. Application to a flexible arm shows the effectiveness of this result, and testing at the Space Research and Design Center (SRDC) at the Naval Postgraduate School emulates a flexible robotics arm freely moving in space.

Although all stability results are proven mathematically, a number of issues remain to be addressed. Specifically, further research is required to choose the optimal number of design parameters for best performance of the proposed system.

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I. INTRODUCTION

A. MOTIVATION AND BACKGROUND

The particular feature, which distinguishes adaptive control systems from other control techniques, is the adaptive loop that provides real time estimates of the Plant Dynamics.

A general structure of an adaptive controller is shown in Figure 1. The system we want to control, called the plant, is modeled in terms of a set of parameters θ^*, unknown or partially known. The goal of the controller is to estimate the plant's parameters and adjust the controller's parameters accordingly.

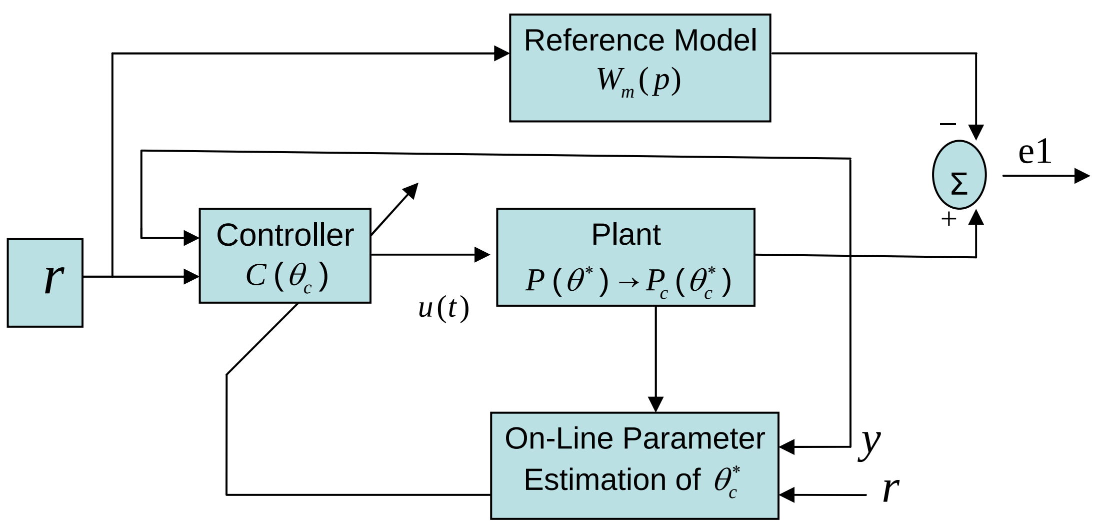

Figure 1.1 Model Reference Adaptive Control.
After [1]

As we can see in Figure 1, there are two loops. One is the standard feedback loop from the output y back to the input u. The other is the loop from the input-output of the plant to the compensator parameters. These two loops can interact with each other and cause instability in the system.

This approach has been around for a number of decades, and stability under ideal conditions is well understood [1].

In spite of the seemingly attractive feature of self-adjusting to time varying dynamics, researchers and engineers have always been skeptical of adaptive control and, as of today, it is not widely used. There are a number of reasons for this skepticism. In particular:

1. Because of internal modulation effects, fast adaptation produces a high frequency input. This input excites unmodeled dynamics easily, and causes systematic instability. Typical examples, from Rohrs [2], have shown that under very mild conditions of unmodeled dynamics that adaptive control system can easily become unstable.
2. There is no guarantee on the bounds of the transient response of both input and output signals. Literature review [2–80] shows that most results do not provide any guidelines on transient performance analysis or explain how to improve transient performance.

During the past few years, there has been greater interest in adaptive control, as evidenced by its prevalence in the literature. A particular approach by Naira Hovakimyan [1] shows that the addition of a filtering term in the loop greatly improves both robustness and transient response behavior. This approach is called "L1 adaptive controller", since it guarantees an arbitrarily small L_∞ bound on the transient response using the small gain theorem on the L_1 norms of the error operators [81]. It was developed for state feedback and then extended to input-output systems. However, this extension does not follow the standard pole placement problem in classical adaptive approaches [82], and therefore, yields in a more general parameterization. This results in an adaptive system for a restrictive class of linear systems, but with the capability of compensating for a wider class of nonlinearities.

The results shown in [81, 83–88] have proved the effectiveness of this approach in terms of robustness and transient behavior.

In this dissertation, we address the problems of designing a class of Adaptive Control systems which yield fast adaptation, thus good transient response, and robustness to modeling errors.

In addition, we show that this approach can be extended to non-minimum phase systems, where the current approaches are unable to stabilize the system. Although this approach requires more knowledge about the system in order to control it, it is still attractive in cases where precise control is needed in the presence of model uncertainties. The typical case we analyze is the precision control of a flexible arm in space.

B. RESEARCH GOALS

After introducing the properties of a general adaptive control system and the Model Reference Adaptive Control (MRAC) system, we examine the drawbacks of these systems and use L1 adaptive control to address them. With the proposed L1 adaptive controller, not only is the tracking error driven to zero, but the transient response is controlled, due to adaptation. The final result is a controller which, under the assumption of low disturbances, yields fast adaptation to parameter variations.

Another goal is to strengthen the robust ability of the adaptive system. After proper choice of low pass filter, reference model, and other important elements, the proposed adaptive controller demonstrates an improvement in the system's ability to deal with the problems of modeling errors, nonlinearities and external bounded disturbances.

C. DISSERTATION ORGANIZATION

This dissertation is organized as follows: Chapter II introduces the fundamentals of adaptive control systems, specifically the direct Model Reference Adaptive Control (MRAC) approach. Rather than repeating the detailed mathematical derivations, which are found in the standard references, a simple first order example is presented, which has all the "flavor" of a full theoretical presentation, but does not include the "fat". Also in this chapter, analytical arguments and computer simulations are used to demonstrate that fast adaptation results in high frequency signals and a possible loss of stability.

Additionally, using this same example, it is shown that the addition of a simple low pass filter in the loops guarantees good transient performance and better robustness to modeling errors.

Chapter III discusses the general extension to a system of arbitrary degree. Typically, the plant in a MRAC system must be minimum phase. This means that the plant to be controlled can be unstable, and that the zeros of its transfer functions must be in the left half side s-plane (in the "stability region"). Additionally, the use of *a priori* knowledge of plant nominal dynamics and its uncertainties is discussed in the design of an adaptive controller, which is globally stable with a transient response having an arbitrarily small bound. This response is obtained provided the effect of external disturbances is negligible.

Chapter IV addresses the minimum phase assumption, where the adaptive system is extended to a class of non-minimum phase systems. It is shown that a non-minimum phase system can be "augmented" to a minimum phase transfer function by adding a properly designed parallel system whose response is zero at steady state. Further, it is shown that if the system is stabilizable by a standard proportional—integral—derivative (PID) controller, then the transfer function of the added system is just the inverse of the PID controller. This is particularly attractive, since a properly design PID controller is generally robust in the presence of model uncertainties. However, such a system has the drawback that its transfer function is not proper, due to the derivative action of the controller. But, since its inverse is implemented in conjunction with the adaptive controller, its transfer function becomes strictly proper, and thus easily realizable. This chapter also discusses an experimental implementation of this approach on a flexible arm in the satellite Research and Design Center (SRDC) at the Naval Postgraduate School (NPS). In this experiment, vibrations due to highly damped flexible modes in the system are properly compensated for so that the position of the end effector quickly settles to the desired position.

Finally, conclusions and further research are presented in the final chapter of this dissertation. Additionally, in order to better illustrate the proposed adaptive algorithm, a number of computer simulations and the results of applications are shown alongside the theoretical presentation.

D. NOTATION

In this dissertation, we define “ p ” the differential operator as

$$px(t) = \dot{x}(t)$$

Also, the notation

$$y(t) = \frac{B(p)}{A(p)} u(t)$$

with $B(p)$, $A(p)$ polynomials in p, indicate the differential equation

$$p^N y(t) + a_1 p^{N-1} y(t) + \cdots + a_N y(t) = b_1 p^{N-1} u(t) + \cdots + b_N u(t)$$

with a_i, b_j the coefficients of $A(p)$, $B(p)$, respectively.

In all situations where the Laplace Transform exists, the differential operator p can be also interpreted as the Laplace variable s.

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II. INTRODUCTION TO ADAPTIVE CONTROL

A. PREFACE

According to Webster's dictionary, to adapt means "to change (oneself), so that one's behavior will conform to new or changed circumstances." This is exactly the goal of an adaptive control system, to self adjust to changing dynamics and environmental conditions. A typical example is the design of a high-performance jet fighter or a fire helicopter [1]. They both operate over a wide range of speeds, altitudes, and weights (before and after loading water for the case of a fire helicopter) and their dynamics are nonlinear and time varying as described by

$$\begin{aligned} \dot{x}(t) &= A(t)x(t) + B(t)u(t) \\ y(t) &= C(t)x(t) + D(t)u(t) \end{aligned} \tag{2.1}$$

where $x(0) = x_0$ is assumed to be the initial state, and $A(t)$, $B(t)$, $C(t)$, and $D(t)$ are functions of the operating time *t*.

As the system goes through different operating conditions, the dynamic model $A(t)$, $B(t)$, $C(t)$, and $D(t)$ changes considerably. The adaptive controller structure consists of a feedback loop and a controller with adjustable gains as shown in Figure 2.1.

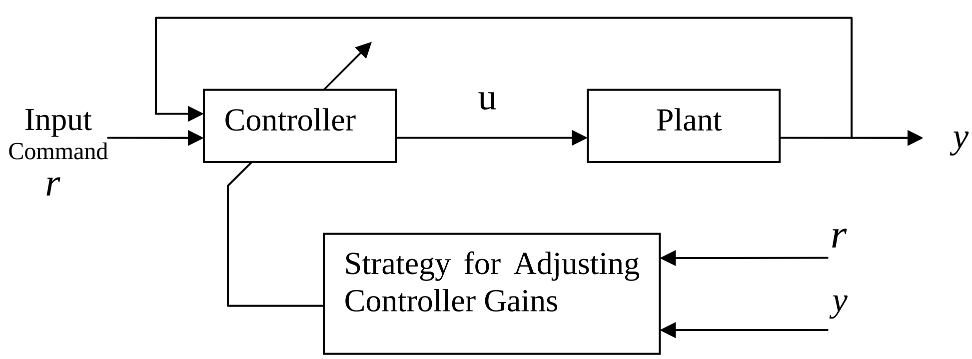

Figure 2.1. Controller Structure with Adjustable Controller Gains. From [1]

In this chapter, two different approaches to adaptive control systems are briefly introduced. They are the Indirect Model Reference Adaptive Control (Indirect MRAC) and the Direct Model Reference Adaptive Control (Direct MRAC) approaches. This is followed by a simple first order example to show the techniques to analyze convergence and stability. Finally, the approach presented in this research is introduced for the first order example to demonstrate the main features of the proposed algorithm.

B. MRC, INDIRECT MRAC, AND DIRECT MRAC

MRAC is derived from the Model Reference Control (MRC) approach. A linear time invariant (LTI), SISO plant of MRC is shown in Figure 2.2.

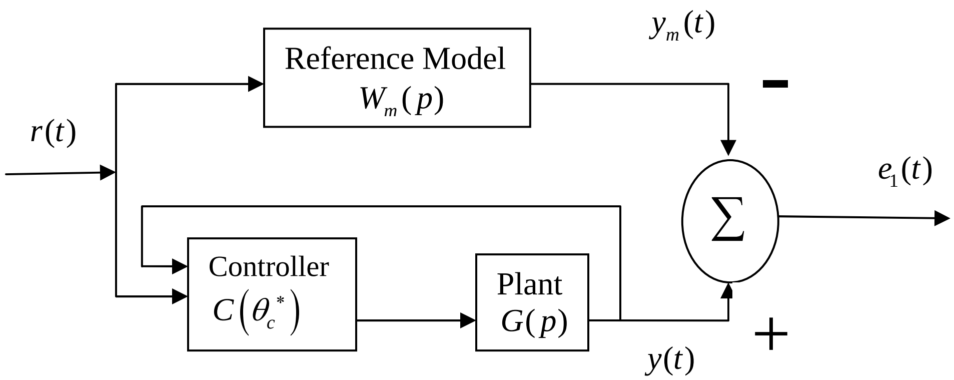

Figure 2.2. Model Reference Control.
From [1]

In MRC, full knowledge of the plant dynamics is assumed. The controller must be designed to meet the designer’s requirements, identified in the reference model, as the desired Input-Output (I/O) properties of the closed loop. The goal of the MRC approach is to find a proper feedback control law so that the closed loop matches a reference model with transfer function $W_m(p)$.

The feedback controller $C\left(\theta_c^*\right)$ is designed so that all signals are bounded and the closed loop transfer function from $r(t)$ to $y(t)$ is equal to $W_m(p)$. This implies that

the object of the desired controller $C\left(\theta_c^*\right)$ is to drive the tracking error $e_1(t) = y(t) - y_m(t)$ close to zero. The requirements of the MRC are as follows:

1. The zeros of the plant must be minimum phase.
2. A good understanding of the plant and performance is required.

If the zeros of the plant are non-minimum phase, the MRC may have an unstable pole zero cancellation thus causing instability. When the plant parameters are not totally known, an adaptive system based on estimates of the plant parameters must be developed. The estimated parameters can be obtained by direct and indirect approaches. The resulting control schemes are known as MRAC. A block diagram of a direct MRAC system is shown in Figure 2.3, and an indirect MRAC system is shown in Figure 2.4.

The difference between a direct MRAC and indirect MRAC is that a direct MRAC updates the estimated controller parameter vector directly from input and output data. In indirect adaptive control, the controller parameters are computed on the basis of the estimated plant dynamics. In this dissertation, the approach is based on the direct MRAC. Accordingly, the concept of the direct MRAC is discussed in the next section.

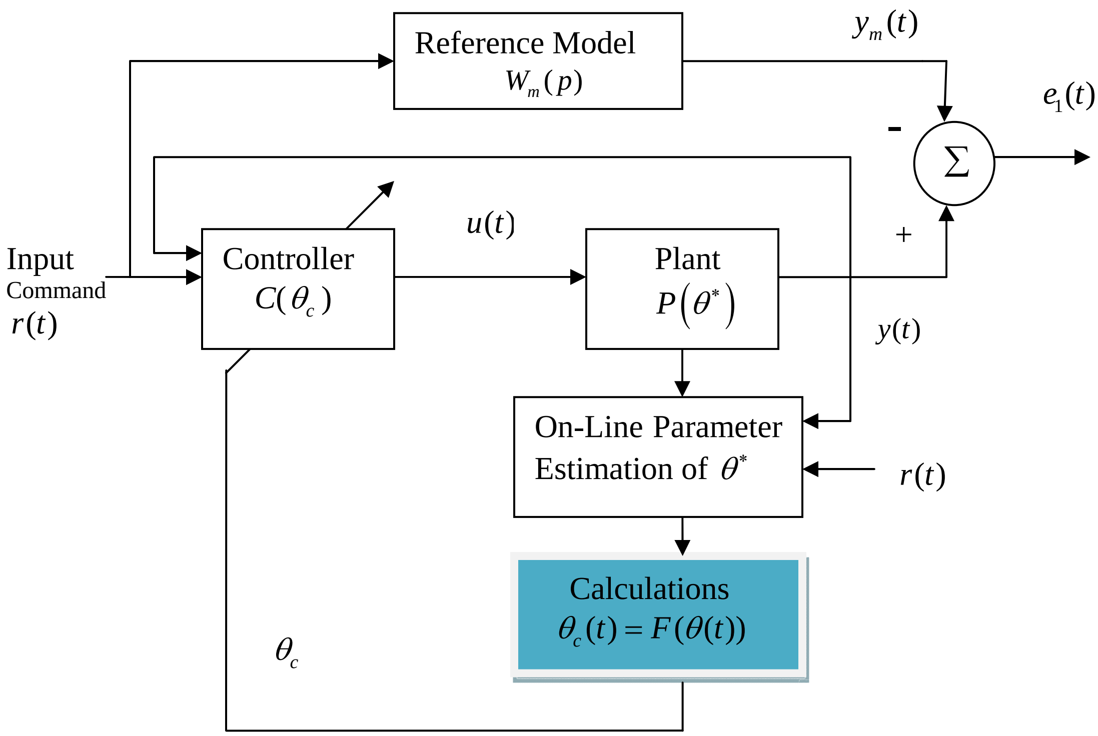

Figure 2.3. Indirect MRAC.
After [1]

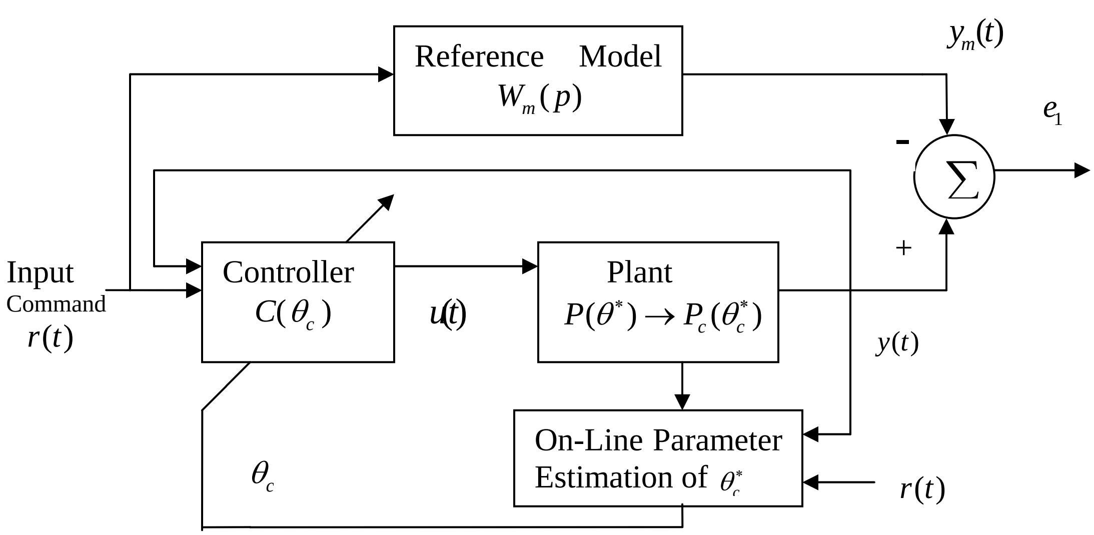

Figure 2.4. Direct MRAC.
From [1]

C. MODEL REFERENCE ADAPTIVE CONTROL

1. Introduction of Model Reference Adaptive Control

The general idea behind MRAC is to create a closed loop controller with estimated parameters that can be updated directly from input/output observations. A standard, simple direct MRAC is shown in Figure 2.5.

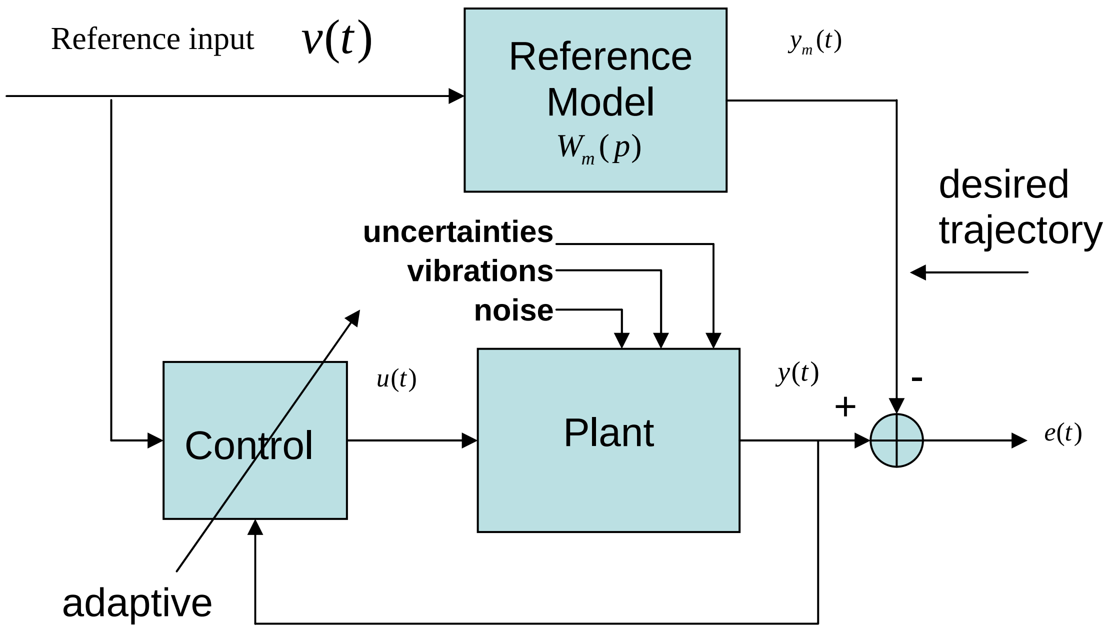

Figure 2.5. MRAC.

As shown in Figure 2.5, the output of the system $y(t)$ is compared to a desired response $y_m(t)$ from a reference model. The control parameters are updated based on the error $e(t)$ between $y(t)$ and $y_m(t)$. The goal of the MRAC is to update the estimated parameters $\hat{\theta}(t)$ so that the plant response matches the response of the reference model $W_m(p)$. In what follows, a first order example of an adaptive control system is presented and the methodology for determining stability and convergence is shown.

2. Theory of Model Reference Adaptive Control

A simple example of a first order case is used to introduce the theory of MRAC. Assume a plant with transfer function

$$W_p(p) = \frac{1}{p-a} \tag{2.1}$$

where a is unknown, and it is not necessarily positive or negative. This implies that no assumption regarding the original plant's stability is made. Equation (2.2) depicts the state space model as

$$\begin{aligned} \dot{x}(t) &= ax(t) + u(t) \\ y(t) &= x(t). \end{aligned} \tag{2.2}$$

Let the reference model have dynamics

$$\begin{aligned} \dot{x}_m(t) &= -a_m x_m(t) + v(t) \\ y_m(t) &= x_m(t) \end{aligned} \tag{2.3}$$

where $a_m > 0$ for stability. The reference output signal $y_m(t)$ represents the desired output the plant has to track. The goal of the controller in MRAC is to drive the tracking error $e(t) = y(t) - y_m(t)$ close to zero. In order to do so, the plant dynamics represented by Equation (2.2) are expressed in terms of the reference model as

$$\dot{x}(t) = -a_m x(t) + (a_m + a)x(t) + u(t). \tag{2.4}$$

Further, the unknown parameter is defined as

$$\theta = a_m + a \tag{2.5}$$

so that Equation (2.4) can be written as

$$\dot{x}(t) = -a_m x(t) + \theta x(t) + u(t). \tag{2.6}$$

This yields the structure of the control input, in terms of the model parameter θ, as

$$u(t) = -\theta x(t) + v(t) \tag{2.7}$$

which yields the desired closed dynamics

$$\dot{x}(t) = -a_m x(t) + v(t). \tag{2.8}$$

If we know the parameter a in equation (2.2), then we can calculate the control parameter in equation (2.5), and use the control input signal as in equation (2.7) to obtain the desired control behavior. However, since we do not know the system dynamics, we use a time varying estimated parameter $\hat{\theta}(t)$ is used to replace the true control parameter θ. In this way, the control becomes

$$u(t) = -\hat{\theta}(t)x(t) + v(t). \tag{2.9}$$

Now, the problem is how to compute the estimated parameter $\hat{\theta}(t)$ based on input/output data. In order to do this, the control input expression of Equation (2.9) is substituted into the plant model Equation (2.6) resulting in.

$$\dot{x}(t) = -a_m x(t) + \tilde{\theta}(t)x(t) + v(t) \tag{2.10}$$

where the parameter error $\tilde{\theta}(t) = \theta - \hat{\theta}(t)$ is the difference between actual value θ and estimated parameter $\hat{\theta}(t)$. Now if the tracking error $e(t)$ is defined as

$$e(t) = y(t) - y_m(t) = x(t) - x_m(t) \tag{2.11}$$

and Equations (2.10) and (2.3) are combined to relate the parameter error $\tilde{\theta}(t)$ with the tracking error $e(t)$, the result is

$$\dot{e}(t) = -a_m e\left(t\right) + \tilde{\theta}(t)x(t). \tag{2.12}$$

In order to provide a viable estimate, the parameter $\hat{\theta}(t)$ is updated as follows

$$\dot{\hat{\theta}}(t) = -\dot{\tilde{\theta}}(t) = \gamma x(t)e(t) \tag{2.13}$$

where $\gamma > 0$ is an arbitrary constant. The basis for this is shown as follows.

Define the Lyapunov function

$$V(e(t),\tilde{\theta}(t)) = \frac{1}{2}|e(t)|^2 + \frac{1}{2}\gamma^{-1}\left|\tilde{\theta}(t)\right|^2 \tag{2.14}$$

Differentiating V along the trajectories of the systems (2.12), (2.13), and (2.14), we obtain

$$\dot{V}(e(t),\tilde{\theta}(t)) = e(t)\left(-a_m e(t) + \tilde{\theta}(t)x(t)\right) + \gamma^{-1}\tilde{\theta}(t)\left(-\gamma x(t)e(t)\right) \tag{2.15}$$

which yields

$$\dot{V}(e(t),\tilde{\theta}(t)) = -a_m |e(t)|^2 \leq 0 \tag{2.16}$$

By standard argument this shows that the estimated parameter is bounded and the error goes to zero [1].

In this example, we can also verify that an upper bound of the error $e(t)$ can be easily determined. In fact, since $V(e(t),\tilde{\theta}(t))$ is non increasing

$$V(e(0),\tilde{\theta}(0)) \geq V(e(t),\tilde{\theta}(t)) \tag{2.17}$$

we can write from (2.14) that

$$\frac{1}{2}|e(0)|^2 + \frac{1}{2}\gamma^{-1}\left|\tilde{\theta}(0)\right|^2 \geq \frac{1}{2}|e(t)|^2 + \frac{1}{2}\gamma^{-1}\left|\tilde{\theta}(t)\right|^2. \tag{2.18}$$

By choosing zero initial condition $x_m(0) = x(0)$, i.e., $e(0) = 0$ we obtain

$$\frac{1}{2}\gamma^{-1}\left|\tilde{\theta}(0)\right|^2 \geq \frac{1}{2}|e(t)|^2 + \frac{1}{2}\gamma^{-1}\left|\tilde{\theta}(t)\right|^2 \geq \frac{1}{2}|e(t)|^2 \tag{2.19}$$

for all $t \geq 0$. This shows an upper bound for the tracking error as

$$|e(t)|^2 \leq \frac{\left|\tilde{\theta}(0)\right|^2}{\gamma} \tag{2.20}$$

for all $t \geq 0$. From Equation (2.20), the relationship between tracking error $e(t)$, adaptive gain γ, and initial parameter error $\tilde{\theta}(0)$ is found. By increasing the gain γ, the maximum error can be made arbitrarily small, thus achieving any desired tracking. Figures 2.6 and 2.7 show simulations for different values of the adaptation gain γ.

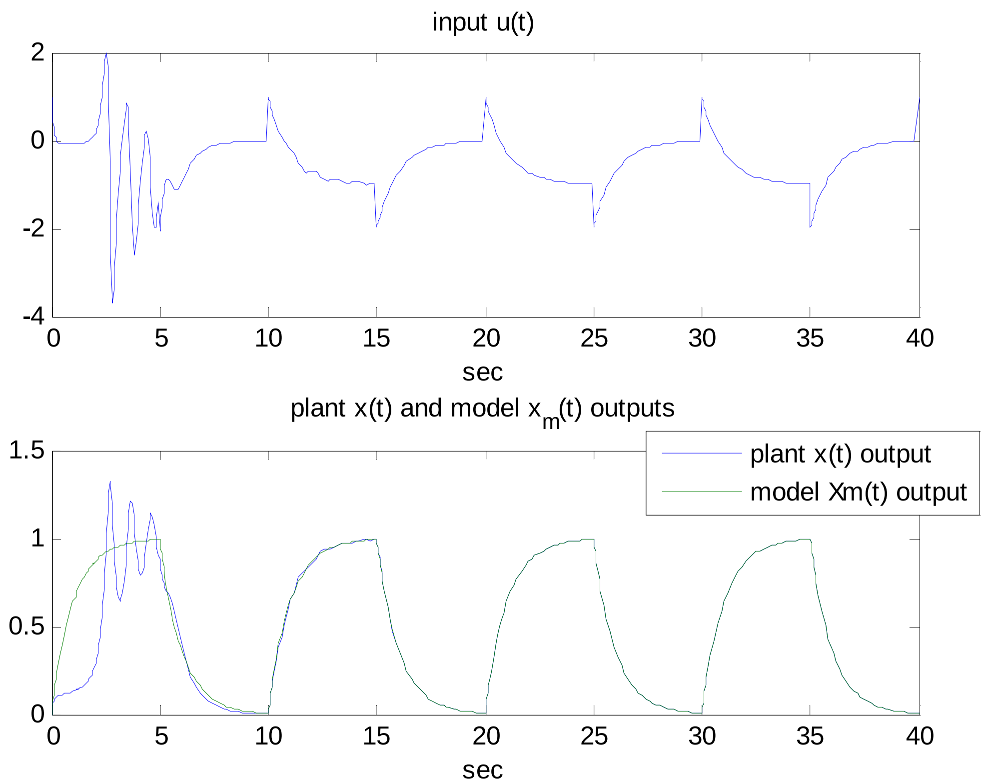

Figure 2.6. Response of Modeled MRAC Adaptive System with $\gamma = 50$.

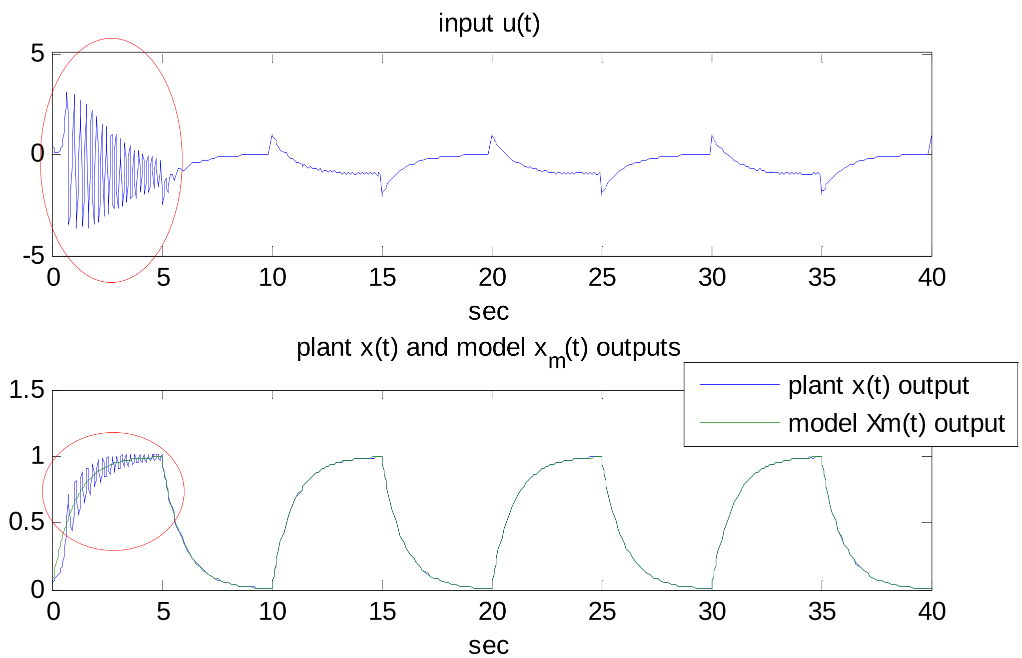

Figure 2.7. Response of Modeled MRAC Adaptive System with $\gamma = 1000$.

These simulations show that, by increasing the adaptation gain γ, the maximum tracking error decreases. However, this comes with a price. As shown in the simulation, the input signal exhibits a higher frequency component. In the next section, these high frequency components will be discussed, including their excitation of unmodeled dynamics, which cause the system to lose tracking or to become unstable. This is seen in Figures 2.8 and 2.9, where the addition of unmodeled poles at $p = -20 + j*5$ and $p = -20 - j*5$, with transfer function $\dfrac{425}{p^2 + 40p + 425}$, causes the system to either becomes unstable or loose tracking (Figure 2.9).

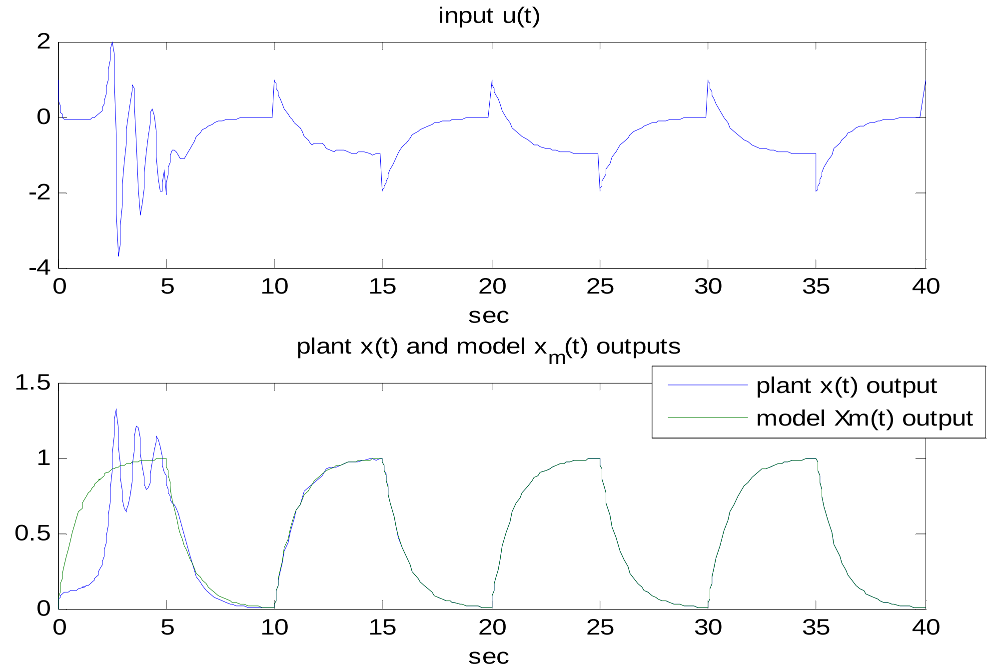

Figure 2.8. Response of Modeled MRAC Adaptive System with $\gamma = 50$.

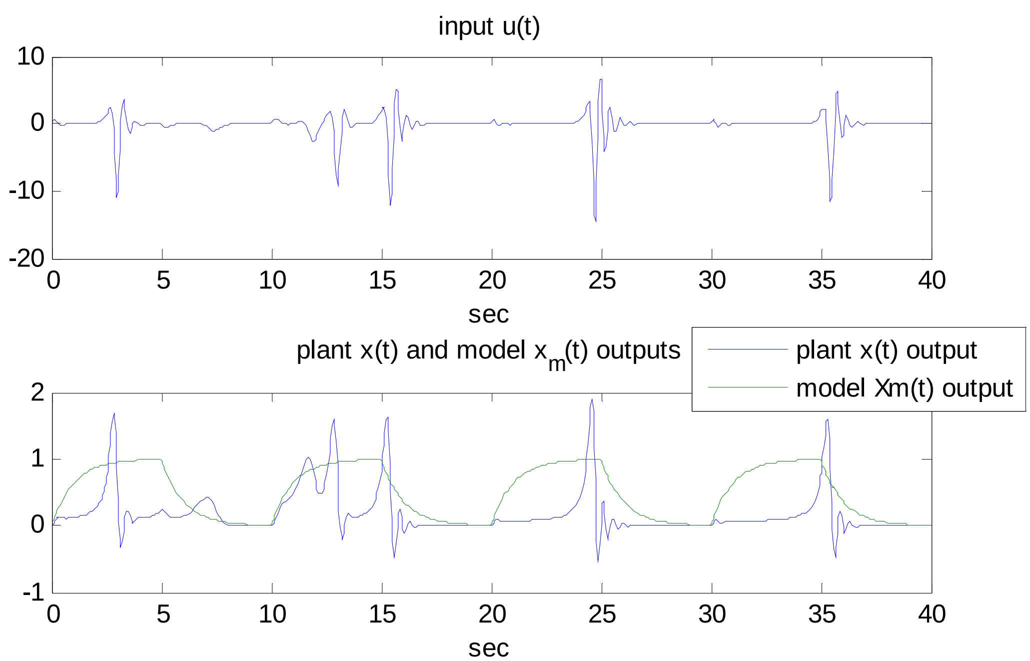

Figure 2.9. Response of MRAC with Unmodeled Dynamics and $\gamma = 50$.

Figure 2.9 shows that, by increasing the adaptation gain γ in the MRAC, the maximum tracking error may not decrease due to the unmodeled dynamics thus leading to loss of tracking.

3. Adaptive Control with a Filtering Action

The arguments presented in Chapter II. C.2 show that there between faster adaptation (large γ) comes at the expenses of a high frequency transient response which can cause instabilities in the presence of modeling errors. This can be seen from the input signal, repeated here for convenience as,

$$u(t) = -\hat{\theta}(t)x(t) + v(t).$$

Using the definition of parameter error $\tilde{\theta}(t) = \theta - \hat{\theta}(t)$ and its relation to the tracking error $e(t)$ shown in Equation (2.12), the input signal can be expressed as

$$u(t) = -\theta x(t) + v(t) + \dot{e}(t) + a_m e(t). \tag{2.21}$$

While the error $e(t)$ can be made arbitrarily small by increasing the adaptation gain γ, its derivative $\dot{e}(t)$ not only is not necessarily small, but it can become quite large due to the high frequencies in the transient response.

A better overall response is obtained by introducing a Low Pass Filter with transfer function $C_F(s)$ at the input signal, which yields

$$u(t) = C_F(p)\left(-\hat{\theta}(t)x(t) + v(t)\right) \tag{2.22}$$

This filter's bandwidth should be larger than the reference model's, so that the filter stops the high frequencies generated by the adaptive system without affecting the dynamics in steady state.

The input signal $u(t)$ can be written again in terms of $\tilde{\theta}(t)$ and $e(t)$ as

$$u(t) \cong C_F(p)\left(-\theta x(t) + v(t)\right) + C_F(p)\left(\dot{e}(t) + a_m e(t)\right). \tag{2.23}$$

The rightmost term shows the effect of the adaptation error, and it can be written as

$$\tilde{u}(t) = (p + a) C_F(p) e(t) \tag{2.24}$$

where the transfer function $(s + a) C_F(s)$ is proper (i.e., no differentiation) and stable. This implies that small values for $e(t)$ also result in small values for $\tilde{u}(t)$, and the controller is always close to an ideal controller with parameter θ.

Figures 2.10 and 2.11 show this behavior. The system is the same as the system presented in Chapter II. C.2, but it is observed that an increased adaptation gain γ yields faster response, with better transient behavior. This adaptive control with a filtering action approach has been called L1 adaptive control [81, 83–88].

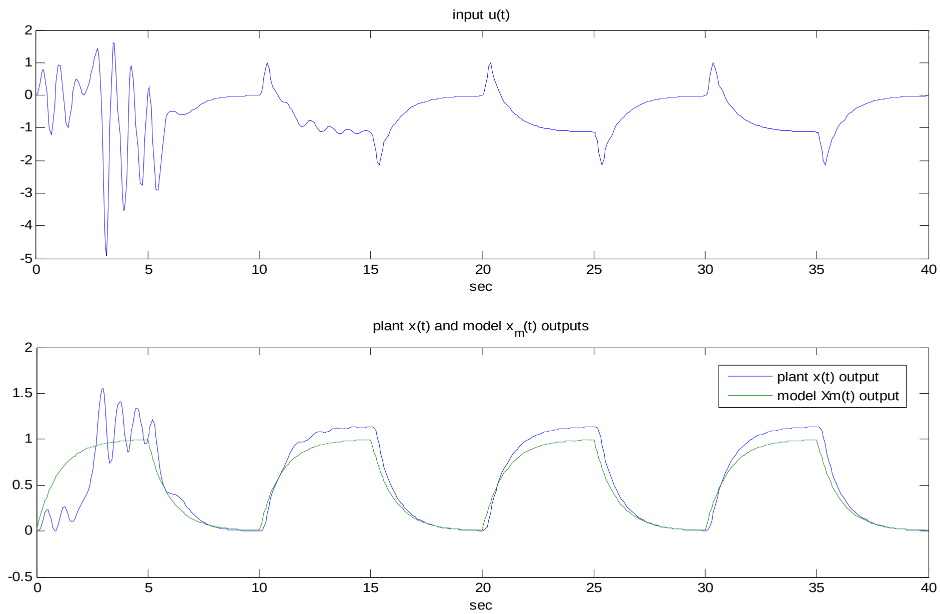

Figure 2.10. Response of L1 Unmodeled MRAC Adaptive System with $\gamma = 50$.

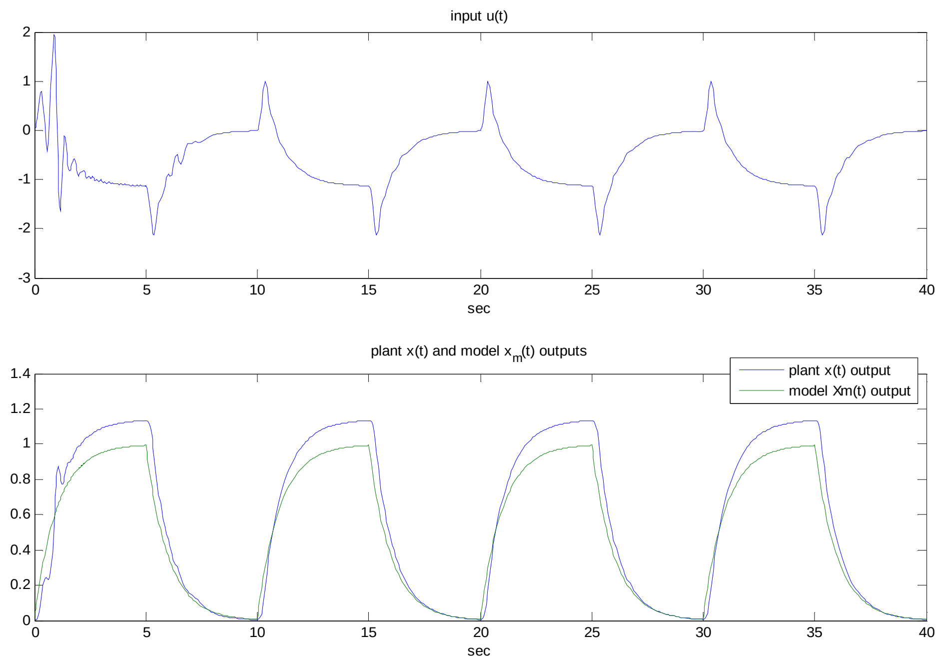

Figure 2.11. Response of L1 Unmodeled MRAC Adaptive System with $\gamma = 1000$.

III. L1 ADAPTIVE CONTROL FOR MINIMUM PHASE SYSTEMS

A. OVERVIEW OF L1 ADAPTIVE CONTROLLER

In this chapter, the adaptive control formulation is extended to a general SISO system. In particular, it is shown that the plant can be parameterized in terms of the controller coefficients, so that they can be estimated directly from the input and output data. Further, the effectiveness of adding a LPF in the controller is addressed. By proper choice of this LPF, it is shown that the magnitude of the transient response due to adaptation can be made arbitrarily small for both input and output signals. The cost is a perturbation of the reference model, which is still globally stable.

This chapter is organized as follows. Model definitions are introduced in Chapter III. B. Parameter estimation is given in Chapter III. C. Then, all properties and elements are combined to show the overall model in Chapter III. D. In Chapter III. E, the L1 adaptive controller is analyzed under ideal conditions, without any disturbances or modeling error. Then, in Chapter III. F, analysis of the ideal case is extended to the conditions of external disturbance and unmodeled dynamics. Finally, in Chapter III. G some examples are considered that compare MRAC systems and verify the theorem of L1 adaptive controller. After this, a deeper exploration of the properties is possible.

B. MODEL DEFINITIONS

In the SISO cases, consider a system with dynamics described by

$$y(t) = k_0 \frac{B(p)}{A(p)} u(t) + w_0(t) \tag{3.1}$$

where
$$A(p) = p^N + a_1 p^{N-1} + ... + a_N \tag{3.2}$$

$$B(p) = p^M + b_1 p^{M-1} + ... + b_M \tag{3.3}$$

are monic. Then the reference model is defined as

$$y_m(t) = \frac{1}{D(p)} v(t) \tag{3.4}$$

where

$$D(p) = p^{N-M} + d_1 p^{N-M-1} + ... + d_{N-M} \tag{3.5}$$

is a Hurwitz polynomial with degree

$$\partial D(p) = \partial A(p) - \partial B(p) \tag{3.6}$$

and $v(t)$ is an arbitrary external bounded input.

In the aforementioned discussion, and in most of the related literature, the assumptions on the plant are as follows:

a. The degree of the numerator $B(p)$, *M*, and the denominator $A(p)$, *N*, are known.

b. The sign of the high frequency coefficient k_0 is known (assume $k_0 > 0$ without loss of generality).

c. The high frequency coefficient k_0 and the parameters $b_1, \ldots, b_M$, $a_1, \ldots, a_N$ of the transfer function of the plant are unknown.

d. The plant can be stable or unstable. However, the numerator $B(p)$ is Hurwitz, i.e., it has roots with negative real parts.

1. Plant Parameterization

Now, the goal is to parameterize the system in terms of the controller parameters. In particular, following the pole placement approach, and assuming the following state space model, as follows:

$$\begin{aligned} \dot{x}(t) &= A_p x(t) + B_p k_0 u(t) \\ y(t) &= C_p x(t) \end{aligned} \tag{3.7}$$

Using an observer and state feedback, the closed loop poles can be arbitrarily placed as shown in

$$\begin{aligned}\dot{\hat{x}}(t) &= (A_p - K_p C_p)\hat{x}(t) + B_p k_0 u(t) + K_p y(t) \\ k_0 u(t) &= -L_p \hat{x}(t) + v(t).\end{aligned} \tag{3.8}$$

If the controller gain is chosen so that

$$\det(sI - A_p + B_p L_p) = B(p)D(p) \tag{3.9}$$

and the observer gain as

$$\det(sI - A_p + K_p C_p) = P(p) \tag{3.10}$$

with $P(p)$ arbitrary, then the Hurwitz closed loop dynamics becomes

$$\begin{aligned}y(t) &= \frac{B(p)}{D(p)B(p)} v(t) \\ &= \frac{1}{D(p)} v(t).\end{aligned} \tag{3.11}$$

In order to determine a suitable plant parameterization, the controller in Equation (3.8) is written as

$$k_0 u(t) = -L_p (sI - A_p + K_p C_p)^{-1} B_p u(t) - L_p (sI - A_p + K_p C_p)^{-1} K_p y(t) + v(t).$$

This expression can also be written as

$$k_0 u(t) = -\frac{k(p)}{P(p)} u(t) - \frac{h(p)}{P(p)} y(t) + v(t) \tag{3.12}$$

where

$$L_p (sI - A_p + K_p C_p)^{-1} B_p = \frac{h(p)}{P(p)} = \frac{h_1 p^{N-1} + \ldots + h_N}{P(p)} \tag{3.13}$$

$$L_p (sI - A_p + K_p C_p)^{-1} K_p = \frac{k(p)}{P(p)} = \frac{k_1 p^{N-1} + \ldots + k_N}{P(p)}\ . \tag{3.14}$$

Define the vectors of coefficients

$$h = [h_1, ..., h_N]^T, \tag{3.15}$$

$$k = [k_1, ..., k_N]^T \tag{3.16}$$

and the filtered signals as

$$\left[\underline{\phi}_u\right] = \frac{1}{P(p)} \begin{bmatrix} p^{N-1} \\ p^{N-2} \\ \vdots \\ p \\ 1 \end{bmatrix} u(t) \tag{3.17}$$

$$\left[\underline{\phi}_y\right] = \frac{1}{P(p)} \begin{bmatrix} p^{N-1} \\ p^{N-2} \\ \vdots \\ p \\ 1 \end{bmatrix} y(t) \tag{3.18}$$

then the plant model can be written as

$$y(t) = \frac{1}{D(p)}(k_0 u(t) + h^T \underline{\phi}_u(t) + k^T \underline{\phi}_y(t)). \tag{3.19}$$

Since the bounds on k_0 are assumed to be known as

$$0 < k_m \le k_0 \le k_M$$

equation (3.19) can be expressed as

$$y(t) = \frac{1}{D(p)}(k_m u(t) + \left(k_0 - k_m\right) u(t) + h^T \underline{\phi}_u(t) + k^T \underline{\phi}_y(t)) \tag{3.20}$$

In summary, the following statement can be made:

Given the plant

$$y(t) = k_0 \frac{B(p)}{A(p)} u(t)$$

and an arbitrary Hurwitz polynomial $D(p)$ with degree $\partial A(p) - \partial B(p)$, the plant can be represented as

$$y(t) = \frac{1}{D(p)}(k_m u(t) + \underline{\theta}^T \phi(t)) \tag{3.21}$$

where

$$\phi(t) = \left[u(t), \frac{p^{N-1}}{P(p)} u(t), \cdots, \frac{1}{P(p)} u(t), \frac{p^{N-1}}{P(p)} y(t), \cdots, \frac{1}{P(p)} y(t) \right] \tag{3.22}$$

and $\theta^T \in R^{2N+1}$ is the set of controller parameters.

It can be easily seen from Equation (3.21) that, in order to track the model $\frac{1}{D(p)}$, the controller has to be of the form

$$k_m u(t) = -\underline{\theta}^T \phi(t) + v(t) \tag{3.23}$$

C. PARAMETER ESTIMATION

In this section, the problem of estimating the parameters $\underline{\theta}$ from the plant dynamics

$$y(t) = \frac{1}{D(p)}(k_m u(t) + \underline{\theta}^T \phi(t)) + w(t) \tag{3.24}$$

is addressed. In order to do so, let $C_E(p)$ be a transfer function such that the $C_E(p)D(p)$ is proper and is of the form

$$C_E(p) = \frac{\omega_E}{(s + \omega_E)^L} \tag{3.25}$$

with $L \geq N - M$. Thus equation (3.24) can be arranged as

$$C_E(p)D(p)y(t) = C_E(p)(k_m u(t) + (k_0 - k_m)u(t) + h^T \phi_u(t) + k^T \phi_y(t)) + w_E(t) \tag{3.26}$$

or

$$C_E(p)D(p)y(t) = C_E(p)(k_m u(t) + \underline{\theta}^T \phi(t)) + w_E(t) \tag{3.27}$$

where $w_E(t) = C_E(p)D(p)w(t)$. If $\overline{w}_E(t)$ is set as the upper bound of $w_E(t)$, then the parameter of equation (3.26) and (3.27) can be written as:

$$C_E(p)D(p)y(t) - k_m c_E(t)u(t) = \theta^T X(t) + w_E(t) \tag{3.28}$$

where $\theta^T \in R^{2N+1}$ and $X(t) = C_E(p)\underline{\phi}(t)$.

Notice that the parameter vector $\underline{\theta}$ appears in (3.28). Therefore, in order to estimate it, call $\hat{\underline{\theta}}(t)$ an estimate and define the error as

$$e(t) = C_E(p)D(p)y(t) - k_m c_E(t)u(t) - \hat{\theta}^T(t)X(t) \tag{3.29}$$

This can be easily related to the parameter error $\tilde{\theta}(t) = \theta - \hat{\theta}(t)$

$$e(t) = \tilde{\theta}^T(t)X(t) + w_E(t). \tag{3.30}$$

Based on this equation, the parameter $\hat{\underline{\theta}}(t)$ can be estimated as follows:

$$\hat{\theta}_i(t) = \mu_i(t)\frac{X_i(t)}{\sigma^2\left|\overline{w}_E(t)\right|^2 + \left\|X(t)\right\|^2}e(t), \quad i = 0, \ldots, 2N \tag{3.31}$$

with

$$\mu_i(t) = \begin{cases} 0, \text{ if } \hat{\theta}_i(t) = \theta_{i,MAX} \text{ and } X_i(t)e(t) > 0 \\ 0, \text{ if } \hat{\theta}_i(t) = \theta_{i,\min} \text{ and } X_i(t)e(t) < 0 \\ \mu \text{, otherwise} \end{cases} \tag{3.32}$$

and μ > 0 arbitrary. By this definition, the estimated parameters $\hat{\theta}_i(t)$ is bounded as shown in Figure 3.1. When the estimated reaches its maximum (minimum) value, adaptation is turned off ($\mu_i(t) = 0$) as long as it tends to increase (decrease).

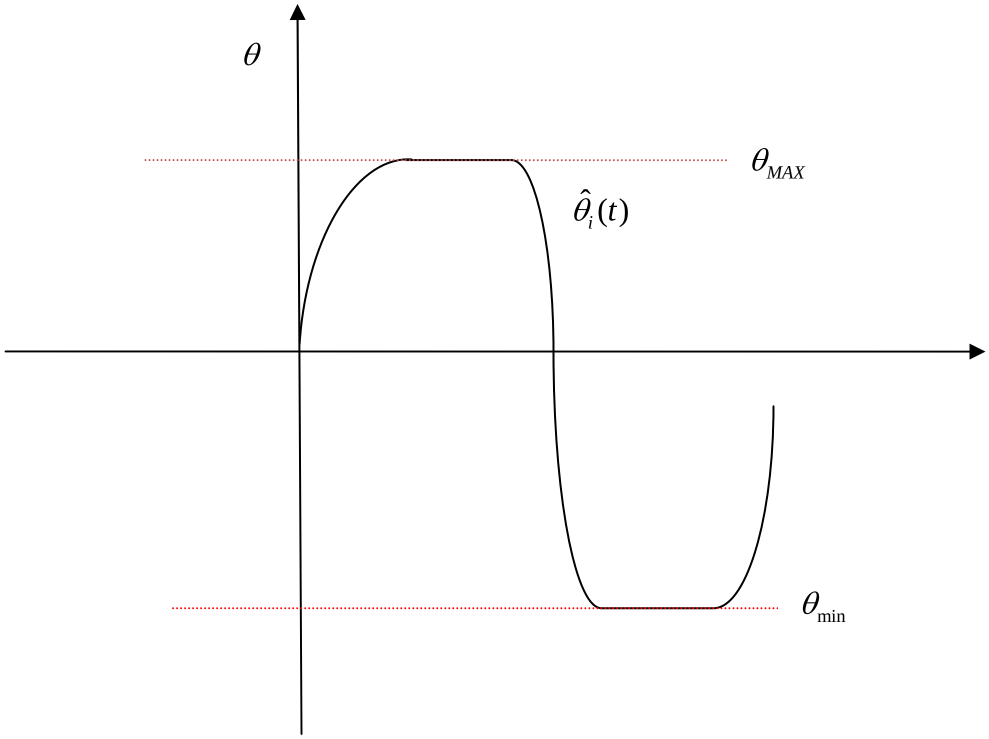

Figure 3.1. The Figure of Estimated Parameter $\hat{\theta}_i(t)$.

From the aforementioned information, it can be shown that for all $t \geq 0$ that

$$
\begin{cases}
\text{a.} \quad \theta_{i,\min} \leq \hat{\theta}_i(t) \leq \theta_{i,MAX} \\
\text{b.} \quad \left|\tilde{\theta}^T(t)X(t)\right| \leq \varepsilon_1(t)\left(\sigma\left|w_E(t)\right| + \left\|X(t)\right\|\right) \\
\text{c.} \quad \left\|\dot{\tilde{\theta}}(t)\right\| \leq \mu\varepsilon_2(t)
\end{cases}
\tag{3.33}
$$

with $\varepsilon_1(t)$, $\varepsilon_2(t)$, such that for all t, T > 0

$$
\int_t^{t+T} \left|\varepsilon_i(\tau)\right|^2 d\tau \le
\begin{cases}
\dfrac{\left\|\tilde{\theta}(t)\right\|^2}{2\mu} & \text{if } w_E(\tau)=0 \text{ for all } \tau \\[2em]
\dfrac{\left\|\tilde{\theta}(t)\right\|^2}{2\mu} + \dfrac{T}{\sigma^2} & \text{otherwise}
\end{cases}
\tag{3.34}
$$

The proof is shown in Appendix A.

In the absence of disturbances, $w_E(t)=0$ for all $t \ge 0$. Therefore, Equation (3.31) can be restated as

$$
\dot{\hat{\theta}}(t) = \mu_i(t)\frac{X_i(t)}{\left\|X(t)\right\|^2}e(t),\ \ i=0,\ldots,2N \tag{3.35}
$$

with

$$
\mu_i(t) = \begin{cases}
0, \text{ if } \hat{\theta}_i(t)=\theta_{i,MAX} \text{ and } X_i(t)e(t)>0 \\[1em]
0, \text{ if } \hat{\theta}_i(t)=\theta_{i,\min} \text{ and } X_i(t)e(t)<0 \\[1em]
\mu\text{, otherwise}
\end{cases}
\tag{3.36}
$$

In the case of no disturbance, in order to avoid singularities during implementation, the denominator $\left\|X(t)\right\|^2$ can be replaced with $a+\left\|X(t)\right\|^2$ for any arbitrary constant a, without loss of generality. In developing the simulated model, the arbitrary constant is set to one, $a=1$. Thus, the denominator becomes $1+\left\|X(t)\right\|^2$.

D. OVERALL MODEL

As in the first order system example presented in Chapter II, the filter $C_F(p)$ is a low pass filter with a DC value of one, $C_F(0)=1$. This filter is critical to the stability and performance of the overall system, and it is addressed in greater detail later in this section, including the criteria for its selection.

Let the input of the adaptive control system be represented as

$$k_m u(t) = C_F(p)\left(-\hat{\theta}^T(t)\phi(t) + v(t)\right) \tag{3.37}$$

where $v(t)$ is the external input and $\phi(t)$ is as previously described in Equation (3.22). The estimated parameters $\hat{\theta}_i(t)$ are computed, as shown in Equation (3.31).

Now the control input in (3.37) is combined with the parameterized plant model in Equation (3.24) to obtain the closed loop dynamics model as

$$\begin{aligned} y(t) &= \frac{C_F(p)}{D(p)}(\tilde{\theta}^T(t)\phi(t) + v(t)) + \frac{(1 - C_F(p))}{D(p)}\left(\theta^T\phi(t)\right) + w(t) \\ k_m u(t) &= C_F(p)\left(\tilde{\theta}^T(t)\phi(t) + v(t)\right) - C_F(p)\left(\theta^T\phi(t)\right) \end{aligned} \tag{3.38}$$

where $\theta^T = \begin{bmatrix} k_0 - k_m & h^T & k^T \end{bmatrix}$ is the true parameters vector. Again $\hat{\theta}(t)$, the estimate, is computed as in Equation (3.31), and $\tilde{\theta}(t) = \theta - \hat{\theta}(t)$ is the parameter error.

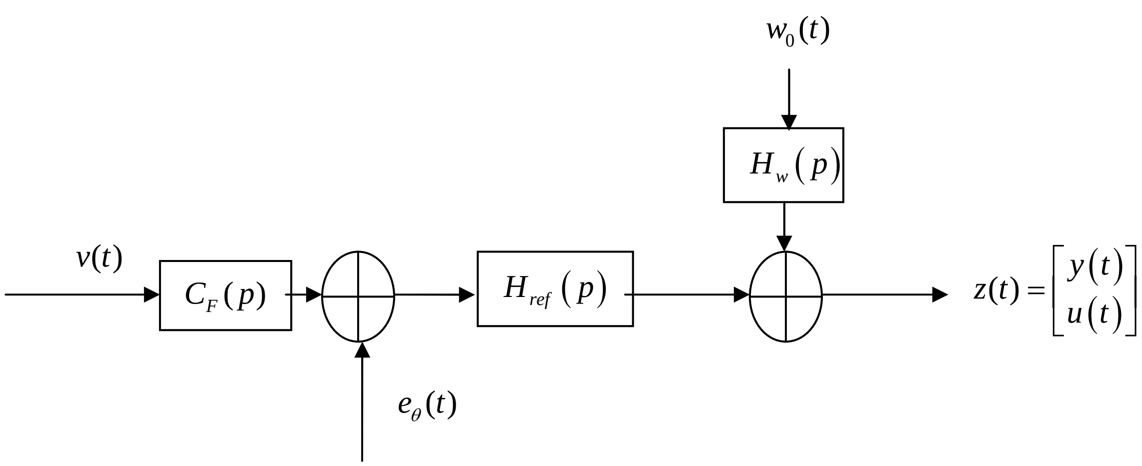

Figure 3.2. Dynamics Model of the Adaptive System.

In order to better understand the global stability of the overall adaptive system, equation (3.38) can be represented by the dynamic model in Figure 3.2 where we define the error term

$$e_\theta(t) = C_F(p)\left(\underline{\tilde{\theta}}^T(t)\underline{\phi}(t)\right) \qquad (3.39)$$

as the effect of parameter error. The overall model $H_{ref}(p)$ is obtained by combining equation (3.38) with the two plant parameterizations

$$y(t) = k_0 \frac{B(p)}{A(p)} u(t)$$

and

$$y(t) = \frac{1}{D(p)}(k_m u(t) + \underline{\theta}^T \phi(t))$$

where $k_m \le k_0 \le k_M$. This yields the dynamics model

$$\begin{bmatrix} y(t) \\ u(t) \end{bmatrix} = H_{ref}(p) C_F(p)\left(\tilde{\theta}^T(t)\phi(t) + v(t)\right) + H_w(p) w_0(t) \qquad (3.40)$$

defined as follows:

$$\begin{aligned} y(t) &= H_{yy}(p) y(t) + H_{yo}(p) C_F(p)\left(\tilde{\theta}^T(t)\phi(t) + v(t)\right) + H_{yw}(p) w_0(t) \\ k_m u(t) &= H_{uy}(p) y(t) + H_{uo}(p) C_F(p)\left(\tilde{\theta}^T(t)\phi(t) + v(t)\right) + H_{uw}(p) w_0(t) \end{aligned} \qquad (3.41)$$

where

$$H_{yy}(p)=\left(1-C_F(p)\right)\left(1-\frac{A(p)}{B(p)D(p)}\right)\left(\frac{1}{1+\gamma C_F(p)}\right)$$

$$H_{y0}(p)=\frac{(1+\gamma)}{\left(1+\gamma C_F(p)\right)D(p)}$$

$$H_{yw}(p)=\frac{\left(1-C_F(p)\right)}{\left(1+\gamma C_F(p)\right)}\left(\frac{A(p)}{B(p)D(p)}+\frac{k(p)}{P(p)D(p)}-1\right)+1-\frac{k(p)}{P(p)D(p)}$$

$$H_{u0}(p)=\frac{1}{1+\gamma C_F(p)}$$

$$H_{uy}(p)=\frac{C_F(p)D(p)}{1+\gamma C_F(p)}\left(\frac{A(p)}{B(p)D(p)}-1\right)$$

$$H_{uw}(p)=\frac{C_F(p)D(p)}{1+\gamma C_F(p)}\left(\frac{A(p)}{B(p)D(p)}+\frac{k(p)}{P(p)D(p)}-1\right)$$

(3.42)

and $\gamma=\frac{k_0}{k_m}-1$ in the interval $0\le\gamma\le\gamma_{MAX}=\frac{k_M}{k_m}-1$. The Proof is shown in Appendix B.

The system $H_{ref}(p)$ in (3.40)–(3.42) represents a perturbation of the dynamic model (3.24) due to the presence of the Low Pass Filter $C_F(p)$. In fact, if we let $C_F(p)\to 1$ for all p, i.e., we assume a filter with infinite bandwidth the terms in (3.42) yield (neglect the disturbance terms for convenience)

$$H_{yy}(p)=\left(1-C_F(p)\right)\left(1-\frac{A(p)}{B(p)D(p)}\right)\left(\frac{1}{1+\gamma C_F(p)}\right)\to 0 \tag{3.43}$$

and

$$H_{y0}(p)=\frac{(1+\gamma)}{\left(1+\gamma C_F(p)\right)D(p)}\to\frac{1}{D(p)}. \tag{3.44}$$

This yields

$$y(t)=\frac{1}{D(p)}(\underline{\tilde{\theta}}^T(t)\phi(t)+v(t)) \tag{3.45}$$

which is the standard dynamic operation in the presence of parameter error $\tilde{\theta}(t)$. For the input term, the analogy is slightly more complicated, but with $C_F(p)=1$, simple algebra yields

$$k_m u(t) = -\hat{\theta}^T(t)\phi(t) + v(t). \tag{3.46}$$

Now the issue is how to choose the filter $C_F(s)$ so that the transformation $H_{ref}(s)$ is exponentially stable. Examination of Equation (3.42) yields two issues:

1. $\left(1+\gamma C_F\left(j\omega\right)\right)^{-1}$ must be stable for all values of γ in the interval $0 \le \gamma \le \gamma_{MAX}$.

2. $\left(I - H_{yy}\left(j\omega\right)\right)^{-1}$ must be stable for all allowable γ, $D(j\omega)$, $B(j\omega)$, and $A(j\omega)$.

This leads to the following two conditions.

Condition 1. Let $C_F\left(p\right)$ be of the form

$$C_F(p) = \left(\frac{\omega_0}{p+\omega_0}\right)\left(\frac{\omega_1{}^L}{\left(p+\omega_1\right)^L}\right) = C_0(p)C_1(p) \tag{3.47}$$

where $L \ge N - M$ the relative degree of the plant. Also, let $\omega_1 > \omega_0$. From the Bode plot, shown in Figure 3.3, it is always possible to choose ω_1 and ω_0 with gain margin larger than the parameter uncertainty γ .

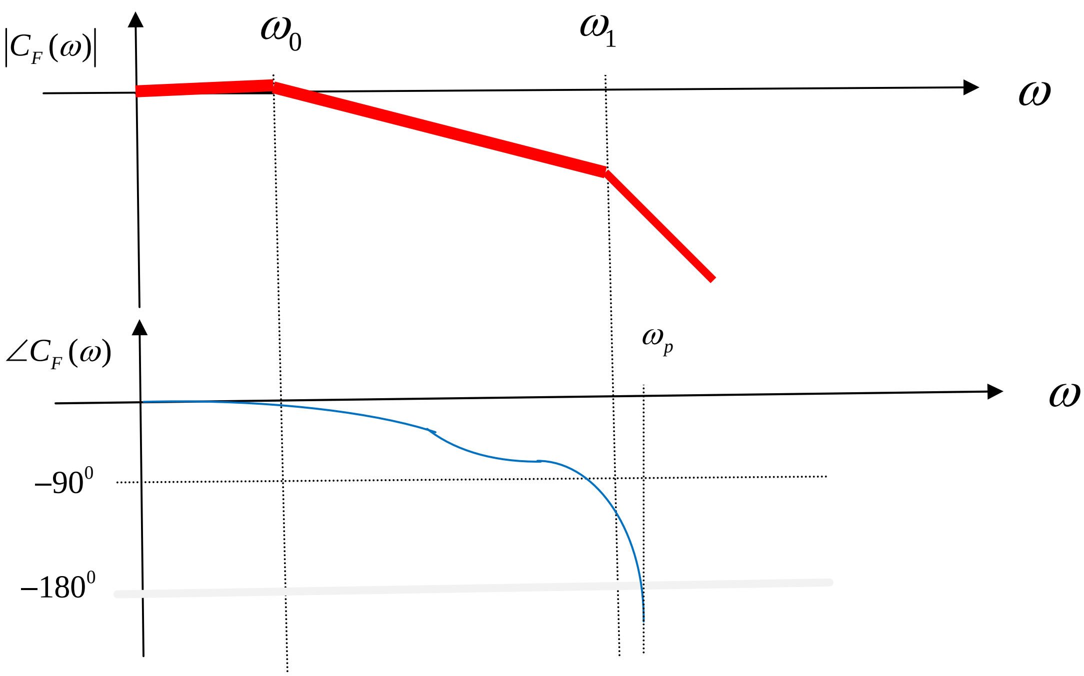

Figure 3.3. Bode Plot of Filter $C_F(p)$.

For the stability of $\left(I - H_{yy}(p)\right)^{-1}$, recall the expression for $H_{yy}(p)$

$$H_{yy}(p) = \left(1 - C_F(p)\right)\left(1 - \frac{A(p)}{B(p)D(p)}\right)\left(\frac{1}{1+\gamma C_F(p)}\right)$$

Since $A(p)$, $B(p)$, and $D(p)$ are all monic polynomials and $B(p)D(p)$ is Hurwitz by assumption, it can be seen that

$$\lim_{p\to\infty}\left(1 - \frac{A(p)}{B(p)D(p)}\right) = 0 \qquad (3.48)$$

Since $A(p)$ and $\left(B(p)D(p)\right)$ have the same degree, this implies that $\left(1 - \frac{A(p)}{B(p)D(p)}\right)$ is low pass. At the same time, with $C_F(s)$ being a low pass filter with unity DC gain, this implies that $\left(1 - C_F(p)\right)$ is a high pass filter. Therefore, the term

$\left(\left(1-C_F\left(p\right)\right)\left(1-\frac{A(p)}{B(p)D(p)}\right)\right)$ can be made arbitrarily small by increasing the bandwidth of $C_F\left(p\right)$. This is summarized by the following condition:

Condition 2. For every positive constant α_M we can always find ω_0 such that

$$\max_{\omega}\left\{\left|\frac{j\omega}{j\omega+\omega_0}\right|\left|1-\frac{A(j\omega)}{B(j\omega)D(j\omega)}\right|\right\}=\alpha_M<1 \tag{3.49}$$

for all ω.

These two conditions are at the basis for choosing the filter

$$C_F(p)=\left(\frac{\omega_0}{p+\omega_0}\right)\left(\frac{\omega_1^{\,L}}{\left(p+\omega_1\right)^L}\right) \tag{3.50}$$

given any arbitrary positive constant $\alpha_M<1$, and choosing ω_0 to satisfy Condition 2, (i.e., Equation (3.49)). Then choosing ω_1, so that the gain margin is

$$\left|C_F\left(j\omega_p\right)\right|_{dB}<\left|1-\alpha_M\right|_{dB}-\left|\gamma_{MAX}\right|_{dB} \tag{3.51}$$

where ω_p is the phase crossover frequency of the filter, guarantees that both Conditions 1 and 2 are satisfied.

E. ADAPTIVE LOOP FEEDBACK FUNCTION H_θ

The parameter error term in Figure again defined as

$$e_\theta(t)=C_F(p)\left(\underline{\tilde{\theta}}^T(t)\underline{\phi}(t)\right)$$

This relates the parameter error $\tilde{\theta}(t)$ with the error $e_\theta(t)$ and it represents a transformation $H_\theta:\phi(t)\to e_\theta(t)$.

Recall the key results concerning parameter estimation previously presented in Chapter III. C. In the case of no external disturbances, the adaptation algorithm yields the following upper bounds

$$\left|\tilde{\theta}^T(t)X(t)\right| \le \left|\varepsilon_1(t)\right|\left\|X(t)\right\| \tag{3.52}$$

and

$$\left\|\dot{\tilde{\theta}}(t)\right\| \le \mu\varepsilon_2(t) \tag{3.53}$$

where

$$X(t) = C_E(p)\phi(t) \tag{3.54}$$

Both terms $\varepsilon_1(t)$ and $\varepsilon_2(t)$ are in L_2 and their magnitudes are upper bounded as

$$\int_0^\infty \left|\varepsilon_i(\tau)\right|^2 d\tau \le \frac{\left\|\tilde{\theta}(0)\right\|^2}{2\mu} \tag{3.55}$$

The goal of this is to show that all the bounds above (3.52)–(3.55) yields an upper of the error $e_\theta(t)$ due to adaptation as

$$\left|e_\theta(t)\right| \le \beta(t)\sup_{0\le\tau\le t}\left\|\phi(\tau)\right\| \tag{3.56}$$

where

$$\lim_{t\to\infty}\beta(t) = 0 \tag{3.57}$$

and

$$\left|\beta(t)\right| \le k_L\left(\frac{\omega_0}{\omega_E}\right)^{\frac{1}{4}} \tag{3.58}$$

where k_L is a constant depending on the relative degree L, ω_0 is the bandwidth of the filter $C_F(p)$, and ω_E is the bandwidth of the filter $C_E(p)$.

This is very important since, going back to Figure 3.2, equation (3.56) shows that the effect of adaptation $e_\theta(t)$ is bounded by a feedback gain $\beta(t)$, which not only goes to zero (3.57), thus making the system globally asymptotically stable, but also its maximum

value (3.58) depends on the bandwidths of the two filters and it can be made arbitrarily small. The rest of this section is devoted to prove this result and, not surprisingly, is very technical in nature.

The abounds (3.56)–(3.58) will be shown as sequence of facts.

Fact 1:

$$\begin{cases} e_\theta(t) = \left(\dfrac{C_1(p)}{C_E(p)}\right) e_\phi(t) = \left(\dfrac{C_1(p)}{C_E(p)}\right) C_0(p)\left(\tilde{e}_{\phi_1}(t) + \tilde{e}_{\phi_2}(t)\right) \\ \left|\tilde{e}_{\phi_1}(t)\right| \le \left|\varepsilon_1(t)\right| \sup\limits_{0\le\lambda\le t} \left\|\phi(\lambda)\right\| \\ \left|\tilde{e}_{\phi_2}(t)\right| \le \left(\left|\varepsilon_1(t)\right| + \mu \displaystyle\int_0^t \left|\varepsilon_2(\tau)\right| g_L\left(\omega_E\left(t-\tau\right)\right) d\tau\right) \sup\limits_{0\le\lambda\le t} \left\|\phi(\lambda)\right\| \end{cases} \tag{3.59}$$

where $\varepsilon_1(t)$ and $\varepsilon_2(t)$ are determined by Equations (3.33) and (3.34), and

$$g_L\left(x\right) = \frac{1}{(L-1)!} \int_0^{+\infty} \tau^{L-1} e^{-\tau} d\tau = \left(\sum_{k=0}^{L-1} \frac{x^k}{k!}\right) e^{-x} \tag{3.60}$$

<u>Proof</u>: First, rearrange Equation (3.39)

$$e_\theta(t) = C_0(p)C_1(p)\left(\tilde{\theta}^T(t)\phi(t)\right) = \left(\frac{C_1(p)}{C_E(p)}\right) C_0(p)\left[C_E(p)\left(\tilde{\theta}^T(t)\phi(t)\right)\right].$$

Then the error can be written as

$$e_\theta(t) = \left(\frac{C_1(p)}{C_E(p)}\right) C_0(p)\left\{\left(\tilde{\theta}^T(t) C_E(p)\phi(t)\right) + \left[C_E(p)\left(\tilde{\theta}^T(t)\phi(t)\right) - \left(\tilde{\theta}^T(t) C_E(p)\phi(t)\right)\right]\right\}$$

If the term in the curly bracket is defined as $e_\phi(t)$, this expression becomes

$$e_\theta(t) = \left(\frac{C_1(p)}{C_E(p)}\right) e_\phi(t). \tag{3.61}$$

The term $e_\phi(t)$ is the sum of two terms

$$e_\phi(t) = e_{\phi_1}(t) + e_{\phi_2}(t)$$

The first term can be written as

$$e_{\phi_1}(t) = C_0(p)\left(\tilde{\theta}^T(t)C_E(p)\phi(t)\right) = C_0(p)\tilde{e}_{\phi_1}(t) \tag{3.62}$$

with

$$\tilde{e}_{\phi_1}(t) = \tilde{\theta}^T(t)C_E(p)\phi(t) = \tilde{\theta}^T(t)X(t). \tag{3.63}$$

Additionally, the second term can be expressed as

$$e_{\phi_2}(t) = C_0(p)\left[C_E(p)\left(\tilde{\theta}^T(t)\phi(t)\right) - \left(\tilde{\theta}^T(t)C_E(p)\phi(t)\right)\right] = C_0(p)\tilde{e}_{\phi_2}(t) \tag{3.64}$$

with

$$\tilde{e}_{\phi_2}(t) = C_E(p)\left(\tilde{\theta}^T(t)\phi(t)\right) - \left(\tilde{\theta}^T(t)C_E(p)\phi(t)\right). \tag{3.65}$$

From equation (3.33) we bound (3.63) as

$$\left|\tilde{e}_{\phi_1}(t)\right| = \left|\tilde{\theta}^T(t)C_E(p)\phi(t)\right| = \left|\tilde{\theta}^T(t)X(t)\right| \le \left|\varepsilon_1(t)\right|\left\|X(t)\right\| \tag{3.66}$$

Since $X(t) = C_E(p)\phi(t)$, the following expression can be written:

$$\left|X(t)\right| \le \int_0^t \left|c_E(t-\tau)\right|\left\|\phi(\lambda)\right\| d\tau \le \left(\int_0^{+\infty}\left|c_E(\tau)\right| d\tau\right) \sup_{0\le\lambda\le t}\left\|\phi(\lambda)\right\|$$

By the way we choose the filter $C_E(p)$ in (3.25), the impulse response $c_E(t) \ge 0$ for all *t*, so the rightmost integral is $C_E(0)$ which is equal to one. Therefore, $X(t)$ can be bounded as follows:

$$\left|X(t)\right| \le \sup_{0\le\lambda\le t}\left\|\phi(\lambda)\right\|. \tag{3.67}$$

Combine this with (3.66) to obtain the upper bound

$$\left|\tilde{e}_{\phi_1}(t)\right| \le \left|\varepsilon_1(t)\right|\left\|X(t)\right\| \le \left|\varepsilon_1(t)\right| \sup_{0\le\lambda\le t}\left\|\phi(\lambda)\right\|. \tag{3.68}$$

This shows the bound of the Fact, in (3.59).

For the bound on $\tilde{e}_{\phi_2}(t)$, expressed in Equation (3.65), it is shown in Appendix C that

$$\tilde{e}_{\phi_2}(t) = -\int_0^t \dot{\tilde{\theta}}^T(\lambda)\left(\int_{t-\lambda}^{t} c_E(\tau)\phi(t-\tau)\,d\tau\right)d\lambda \tag{3.69}$$

The derivative $\left|\dot{\tilde{\theta}}(t)\right|$ is upper bounded as in (3.53), so that (3.69) above becomes

$$\left|\tilde{e}_{\phi_2}(t)\right| \le \mu\int_0^t \left|\varepsilon_2(\lambda)\right|\left(\int_{t-\lambda}^{+\infty} c_E(\tau)\,d\tau\right)d\lambda \sup_{0\le\tau_0\le t}\left\|\phi(\tau_0)\right\| \tag{3.70}$$

where again $c_E(t) \ge 0$ is the impulse response of the filter $C_E(p) = \left(\dfrac{\omega_E{}^L}{\left(p+\omega_E\right)^L}\right)$. Then

$$\int_{t-\lambda}^{+\infty} c_E(\tau)d\tau = \int_{t-\lambda}^{+\infty} \frac{\omega_E^L \tau^{L-1}}{(L-1)!}e^{-\omega_E \tau}d\tau = g_L\left(\omega_E\left(t-\tau\right)\right) \tag{3.71}$$

This shows the second inequality in (3.59), and Fact 1 is proved.

QED

<u>Fact 2</u>: The error signal

$$e_\theta(t) = \left(\frac{C_1(p)}{C_E(p)}\right)e_\phi(t) \tag{3.72}$$

can be upper bounded as

$$\left|e_\phi(t)\right| \le \beta(t)\sup_{0\le\tau\le t}\left\|\phi(\tau)\right\| \tag{3.73}$$

where

$$\lim_{t\to\infty}\beta(t) = 0$$

$$\left|\beta(t)\right| \le \left(A_L\sqrt{\frac{\mu}{2\omega_E}} + \frac{1}{2}\sqrt{\frac{\omega_0}{\mu}}\right)\left\|\tilde{\theta}(0)\right\|, \text{ for all } t\ge 0 \tag{3.74}$$

and

$$A_L = \left(\int_0^{+\infty} \left| \sum_{k=0}^{L-1} \frac{x^k}{k!} \right|^2 e^{-2x} dx \right)^{\frac{1}{2}} = \left(\sum_{k_1=0}^{L-1} \sum_{k_2=0}^{L-1} \frac{1}{2^{k_1+k_2+1}} \frac{(k_1+k_2)!}{k_1!k_2!} \right)^{\frac{1}{2}}.$$

<u>Proof</u>: From Equation (3.59)

$$e_\theta(t) = \left(\frac{C_1(p)}{C_E(p)} \right) e_\phi(t)$$

with

$$e_\phi(t) = e_{\phi_1}(t) + e_{\phi_2}(t)$$

Using the bounds in Equation (3.59) yields

$$\left| e_{\phi_1}(t) \right| \le \beta_1(t) \sup_{0 \le \lambda \le t} \left\| \phi(\lambda) \right\| \tag{3.75}$$

Where, using Schwartz inequality,

$$\left| \beta_1(t) \right| = \int_0^t \omega_0 e^{-\omega_0(t-\tau)} \left| \varepsilon_1(\tau) \right| d\tau \le \omega_0 \left(\int_0^{+\infty} e^{-2\omega_0 \tau} d\tau \right)^{\frac{1}{2}} \left(\int_0^{+\infty} \left| \varepsilon_1(\tau) \right|^2 d\tau \right)^{\frac{1}{2}}. \tag{3.76}$$

This implies that

$$\left| \beta_1(t) \right| \le \left(\frac{{\omega_0}^2}{-2\omega_0} e^{-2\omega_0 \tau} \Big|_0^\infty \right)^{\frac{1}{2}} \left(\frac{\left\| \tilde{\theta}(0) \right\|}{2\mu} \right)^{\frac{1}{2}} \tag{3.77}$$

And, therefore

$$\left| \beta_1(t) \right| \le \frac{\left\| \tilde{\theta}(0) \right\|}{2} \sqrt{\frac{\omega_0}{\mu}} \tag{3.78}$$

for all $t \ge 0$. Furthermore, since $\varepsilon_1(t) \in L_2$, then

$$\lim_{t \to \infty} \beta_1(t) = 0 \tag{3.79}$$

Therefore, Equation (3.75) can be rewritten as

$$\left| e_{\phi_1}(t) \right| \le \frac{\left\| \tilde{\theta}(0) \right\|}{2} \sqrt{\frac{\omega_0}{\mu}} \sup_{0 \le \lambda \le t} \left\| \phi(\lambda) \right\|$$

For the second part of $e_\theta(t)$, recall from Equation (3.59)

$$\left|\tilde{e}_{\phi_2}(t)\right| \le \tilde{\beta}_2(t) \sup_{0\le\tau\le t} \|\phi(\tau)\| \tag{3.80}$$

with

$$\tilde{\beta}_2(t) = \mu\varepsilon_2(t) * g_L(\omega_E t). \tag{3.81}$$

Since $\varepsilon_2(t) \in L_2$, then

$$\lim_{t\to\infty} \tilde{\beta}_2(t) = 0.$$

Again using Schwartz inequality in the convolution integral in (3.81), we obtain an upper bound for $\tilde{\beta}_2(t)$

$$\left|\tilde{\beta}_2(t)\right| \le \mu\left(\int_0^{+\infty} \left|\varepsilon_2(\lambda)\right|^2 d\lambda\right)^{\frac{1}{2}} \left(\int_0^{+\infty} \left|g_L(\omega_E\lambda)\right|^2 d\lambda\right)^{\frac{1}{2}} \le A_L \cdot \sqrt{\frac{\mu}{2\omega_E}} \left\|\tilde{\theta}(0)\right\|. \tag{3.82}$$

Since $c_0(t) = \omega_0 e^{-\omega_0 t}$, for $t \ge 0$, it is easy to see that

$$\beta_2(t) = c_0(t) * \tilde{\beta}_2(t) \tag{3.83}$$

still yields the limit

$$\lim_{t\to\infty} \beta_2(t) = 0 \tag{3.84}$$

and the same upper bound

$$\left|\beta_2(t)\right| \le \mu\left(\int_0^{+\infty} \left|\varepsilon_2(\lambda)\right|^2 d\lambda\right)^{\frac{1}{2}} \left(\int_0^{+\infty} \left|g_L(\omega_E\lambda)\right|^2 d\lambda\right)^{\frac{1}{2}} \le A_L \sqrt{\frac{\mu}{2\omega_E}} \left\|\tilde{\theta}(0)\right\|. \tag{3.85}$$

Combining Equations (3.78) and (3.85) yields the result in Fact 2.

QED

As noted, the dimension of the adaptive gain, $\mu > 0$, is inverse time or $time^{-1}$, as demonstrated in Equation (3.31). As a consequence, $\frac{\mu}{\omega}$ in Equation (3.74) is dimensionless.

The bound on the term $e_\phi(t)$ can be found in Equations (3.73) and (3.74) as

$$\left|e_\phi(t)\right| \le \beta(t) \sup_{0\le\tau\le t} \|\phi(\tau)\| \tag{3.73}$$

$$|\beta(t)| \le \left(A_L \sqrt{\frac{\mu}{2\omega_E}} + \frac{1}{2}\sqrt{\frac{\omega_0}{\mu}} \right) \left\| \tilde{\theta}(0) \right\|, \text{ for all } t \ge 0 \tag{3.74}$$

From Equations (3.73) and (3.74), $e_\phi(t)$ can be minimized by choosing the adaptive gain μ in Equation (3.31) in terms of the parameters ω_0, ω_E, and degree L of the filter $C_F(s)$. And then, by straightforward differentiation the smallest upper bound is obtained

$$\mu = \frac{1}{\sqrt{2}A_L}\sqrt{\omega_0\omega_E} \tag{3.86}$$

which yields the bound on the adaptive transformation $H_\theta(p)$

$$|e_\phi(t)| \le k_L \left(\frac{\omega_0}{\omega_E} \right)^{\frac{1}{4}} \sup_{0\le\tau\le t} \|\phi(\tau)\| \tag{3.87}$$

for some constant k_L, dependent only on the filter's order and degree, $L \ge N - M$.

This result shows how the upper bound on the transient response $e_\theta(t)$ due to adaptation depends in a fairly simple fashion on the ratio of the bandwidths ω_0 and ω_E of the two filters $C_F(s)$ and $C_E(s)$ in the control and the estimation loops. As $\left(\frac{\omega_0}{\omega_E} \right) \to 0$ the effect of adaptation in both input and output signals tends to zero

In the next section, we show this result formally, based on the what has been shown in this section, summarized below:

$$e_\theta(t) = \frac{C_1(p)}{C_E(p)} e_\phi(t) \tag{3.88}$$

it can be shown that $e_\theta(t)$ has the same bound as $e_\phi(t)$

$$\begin{cases} |e_\theta(t)| \le \beta(t) \sup_{0\le\tau\le t} \|\phi(\tau)\| \\ |\beta(t)| \le k_L \left(\dfrac{\omega_0}{\omega_E}\right)^{\frac{1}{4}} \\ \lim_{t\to\infty} \beta(t) = 0 \end{cases} \tag{3.89}$$

F. STABILITY AND BOUNDEDNESS OF THE ADAPTIVE SYSTEM

In this section, the concepts up to this point are combined and culminate in the main result of this dissertation.

First define the vector $\underline{z}(t)$

$$\underline{z}(t) = \begin{bmatrix} y(t) \\ u(t) \end{bmatrix} \tag{3.90}$$

Assuming zero disturbances, from (3.40) and (3.89) we obtain

$$\begin{cases} \underline{z}(t) = H_{ref}(p)\big(e_\theta(t) + C_F(p)v(t)\big) \\ e_\theta(t) = H_\theta(p)\big(\underline{\phi}(t)\big) \end{cases} \tag{3.91}$$

where $H_{ref}(p)$ is exponentially stable, and $H_\theta(p)$ is bounded by $\beta(t)$ in Equation (3.89). Also, recall that $\underline{\phi}(t)$ is a vector of filtered input and output signals, which can be written as

$$\underline{\phi}(t) = \underline{\Phi}(p)\underline{z}(t) \tag{3.92}$$

where $\underline{\Phi}(p)$ is exponentially stable transformation in (3.17) and (3.18).

The whole system, obtained by combining Equations (3.91) and (3.92), is shown in Figure 3.4.

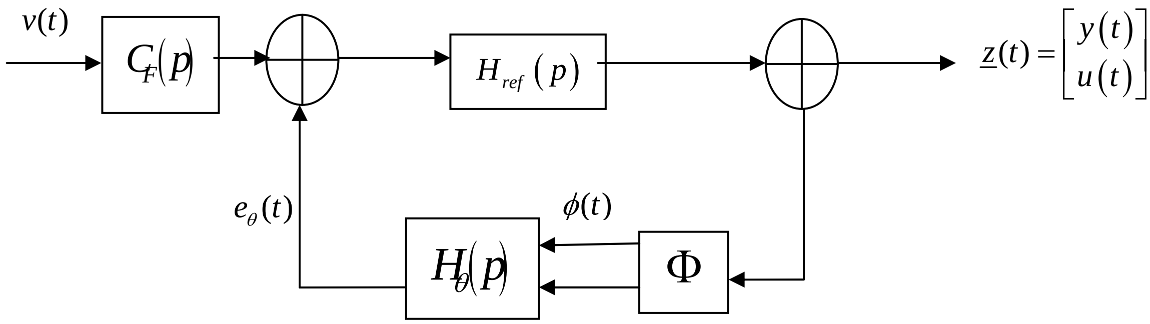

Figure 3.4. Dynamics Model of the Adaptive System with No Disturbance.

Since, $\beta(t)\to 0$ (Equation (3.89)), and $H_{ref}(p)$ and $\underline{\Phi}(p)$ are exponentially stable, the closed loop system is exponentially stable and all signals are bounded.

Also, if the reference input and output signal are defined as

$$z_{ref}(t)=H_{ref}(p)C_F(p)v(t) \tag{3.93}$$

and $\tilde{z}(t)=z(t)-z_{ref}(t)$, then

$$\begin{cases} \tilde{z}(t)=H_{ref}(p)e_\theta(t) \\ e_\theta(t)=H_\theta(p)\Phi(p)\left(\tilde{z}(t)+z_{ref}(t)\right) \end{cases} \tag{3.94}$$

Each system, $H_{ref}(p)$, $H_\theta(p)$, and $\Phi(p)$, is Bounded Input and Bounded Output (BIBO) stable and all terms can be bounded as

$$\|\tilde{z}\|_\infty \le |H_{ref}(p)|_\infty\,|e_\theta|_\infty\,, \tag{3.95}$$

$$\|\phi\|_\infty \le |\Phi(p)|_\infty\,\|z\|_\infty\,, \tag{3.96}$$

$$\|z\|_\infty \le \|\tilde{z}\|_\infty+\|z_{ref}\|_\infty\,, \tag{3.97}$$

and

$$|e_\theta|_\infty \le |H_\theta(p)|_\infty\,|\Phi(p)|_\infty\left(\|\tilde{z}\|_\infty+\|z_{ref}\|_\infty\right). \tag{3.98}$$

From these, it is easy to see that

$$\left\|\tilde{z}\right\|_\infty \le \left|H_{ref}(p)\right|_\infty \left|H_\theta(p)\right|_\infty \left|\Phi(p)\right|_\infty \left(\left\|\tilde{z}\right\|_\infty + \left\|z_{ref}\right\|_\infty\right) \tag{3.99}$$

which yields

$$\left\|\tilde{z}\right\|_\infty \le (1-K)^{-1} K \left\|z_{ref}\right\|_\infty \tag{3.100}$$

with

$$K = \left|H_{ref}(p)\right|_\infty \left|H_\theta(p)\right|_\infty \left|\Phi(p)\right|_\infty \tag{3.101}$$

Inequality (3.100) is valid only when the gain *K* is smaller than one as $0 < K < 1$.

From the results above, recall

$$\left|H_\theta(p)\right|_\infty = \sup_{0\le t\le\infty} \left|\beta(t)\right| \le k_L \left(\frac{\omega_0}{\omega_E}\right)^{\frac{1}{4}} \tag{3.102}$$

where k_L depends on the filter order only. Therefore, it is observed that as $\frac{\omega_0}{\omega_E} \to 0$, the overall loop gain *K* goes to zero and, therefore, the error terms

$$\tilde{z}(t) = \begin{vmatrix} y(t) - y_{ref}(t) \\ u(t) - u_{ref}(t) \end{vmatrix} \tag{3.103}$$

are such that

$$\lim_{\frac{\omega_0}{\omega_E}\to\infty} \sup_{0\le t\le+\infty} \left|y(t) - y_{ref}(t)\right| = 0 \tag{3.104}$$

and

$$\lim_{\frac{\omega_0}{\omega_E}\to\infty} \sup_{0\le t\le+\infty} \left|u(t) - u_{ref}(t)\right| = 0. \tag{3.105}$$

In other words, the maximum value of the transient response due to the adaptive controller tends to become arbitrary small.

In the next section, we show that computer simulations support the results presented above.

G. EXAMPLES OF APPLICATION

1. L1 Adaptive Control in Ideal Case (No Disturbance)

Consider an unstable plant

$$y(t) = k_0 \frac{1}{p-1} u(t) \tag{3.106}$$

where $1 \le k_0 \le 4$, and the reference model

$$y_m(t) = \frac{1}{p+2} v(t). \tag{3.107}$$

Then

$$B(p) = 1 \;, A(p) = p - 1, \; D(p) = p + 2. \tag{3.108}$$

First, the true coefficients are determined, assuming $k_0 = 3$ as the true value of the parameter. From $A(p)$ and $B(p)$, the related space state equation can be determined and is expressed as follow:

$$\begin{aligned} \dot{x}(t) &= A_p x(t) + B_p u(t) \\ y(t) &= C_p x(t) \end{aligned} \tag{3.109}$$

where $$\begin{cases} A_p = 1 \\ B_p = 1 \\ C_p = 3 \end{cases}$$

Following the steps mentioned above, the closed loop system is obtained

$$\begin{aligned} \dot{x}(t) &= (A_p - B_p L_p) x(t) + B_p v(t) \\ y(t) &= C_p x(t) \end{aligned} \tag{3.110}$$

where controller gain $L_p = 3$ is derived from Equation (3.9).

Let the observer be

$$\begin{aligned}\dot{\hat{x}}(t) &= A_p \hat{x}(t) + B_p u(t) + K_p (y(t) - \hat{y}(t)) \\ \hat{y}(t) &= C_p \hat{x}(t)\end{aligned} \tag{3.111}$$

where $P(s) = p + 2$ and the observer gain $K_p = 1$ is derived from Equation (3.10).

From Equations (3.15), (3.16), (3.22), and (3.23), the system can be represented as

$$k_0 u(t) = -\left[\left(k_0 - k_m \right) \quad h^T \quad k^T \right] \underline{\phi}(t) + v(t) = -\underline{\theta}^T \underline{\phi}(t) + v(t) \tag{3.112}$$

where
$$\begin{cases} \underline{\theta}^T = \left[2 \quad 3 \quad 4.5 \right]^T \\ \phi(t) = \left[u(t) \quad \dfrac{1}{s+2} u(t) \quad \dfrac{1}{s+2} y(t) \right]^T . \end{cases} \tag{3.113}$$

Now, with $k_0 = 3$, a sufficiently large value for ω_0 must be chosen. Additionally, according to **Condition 1** and Equation (3.49), the magnitude of the response of $\left(1 - \dfrac{A(p)}{B(p)D(p)} \right)$ must be less than 1. The magnitude of the Bode Plot of $\left(1 - \dfrac{A(p)}{B(p)D(p)} \right)$ is shown in Figure 3.5 and it is smaller than one (negative dB's) for all frequencies larger than than 2.5 rad/sec. This can be used as a guideline to determine the bandwidth ω_0 of the filter $C_F(s)$.

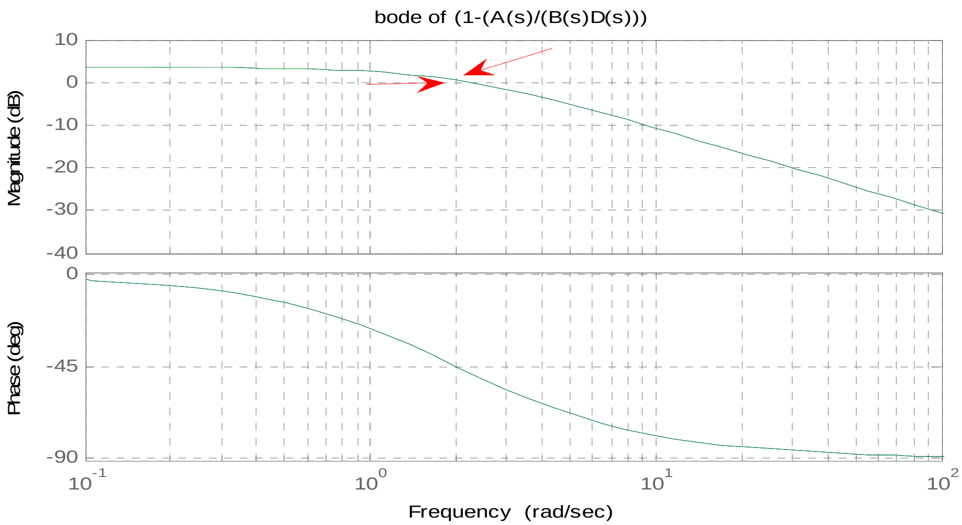

Figure 3.5. The Bode Plot of $\left(1-\frac{A(p)}{B(p)D(p)}\right)$.

With a choice of $\omega_0 = 5$, the LPF becomes $C_F(p) = \left(\frac{\omega_0}{p+\omega_0}\right) = \frac{5}{s+5}$ and we must verify that **Condition 2** is satisfied. Recall this condition

$$\max_{\omega} \max_{0 \le \gamma \le \gamma_{MAX}} \left| H_{yy}(j\omega) \right| < 1$$

where

$$H_{yy}(p) = \left(1 - C_F(p)\right)\left(1 - \frac{A(p)}{B(p)D(p)}\right)\left(\frac{1}{1+\gamma C_F(p)}\right)$$

For a number of different realizations of γ, $H_{yy}(p)$ is plotted, as shown in Figure 3.6. The simulation results conform to the requirement of **Condition 2**.

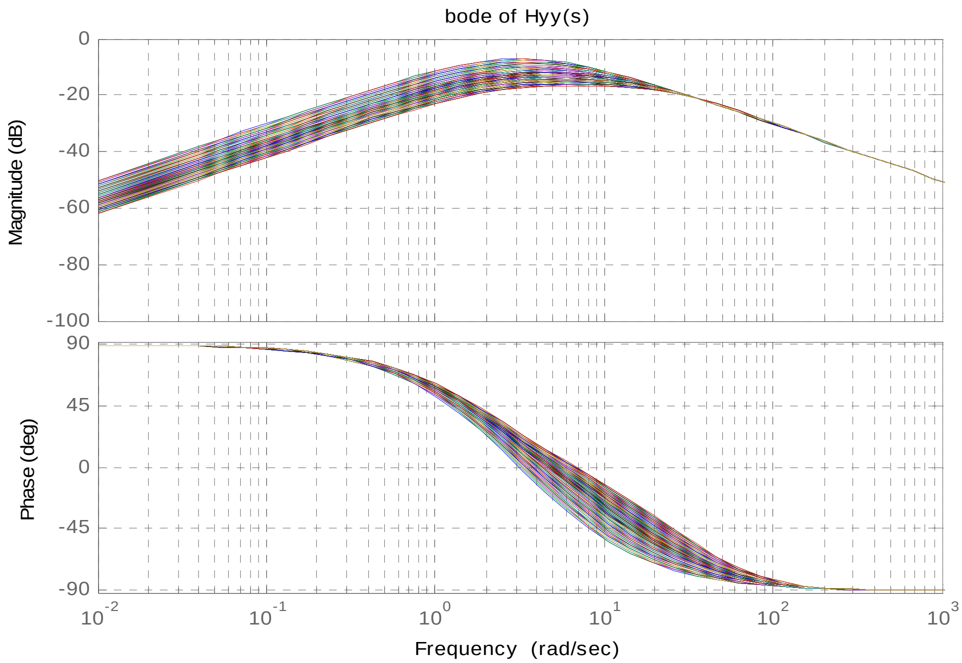

Figure 3.6. The Bode Plot of $H_{yy}(p)$ for 100 Plant Realizations.

Next, if ω_E is chosen such that $\omega_E > \omega_0$, and the degree L of $C_E(p)$ is guaranteed to meet $L \geq N - M$, then $C_E(p)D(p)$ will be proper. In this example, let $\omega_E = 100$ rad/sec, $L = 1 = M - N$, and, therefore,

$$C_E(p) = \frac{\omega_E{}^L}{(p+\omega_E)^L} = \frac{100}{(p+100)}. \tag{3.114}$$

Since this case is implemented with no disturbances, $w_E(t) = 0$. Figure 3.7 and Figure 3.8 depict simulation results of the case without disturbances.

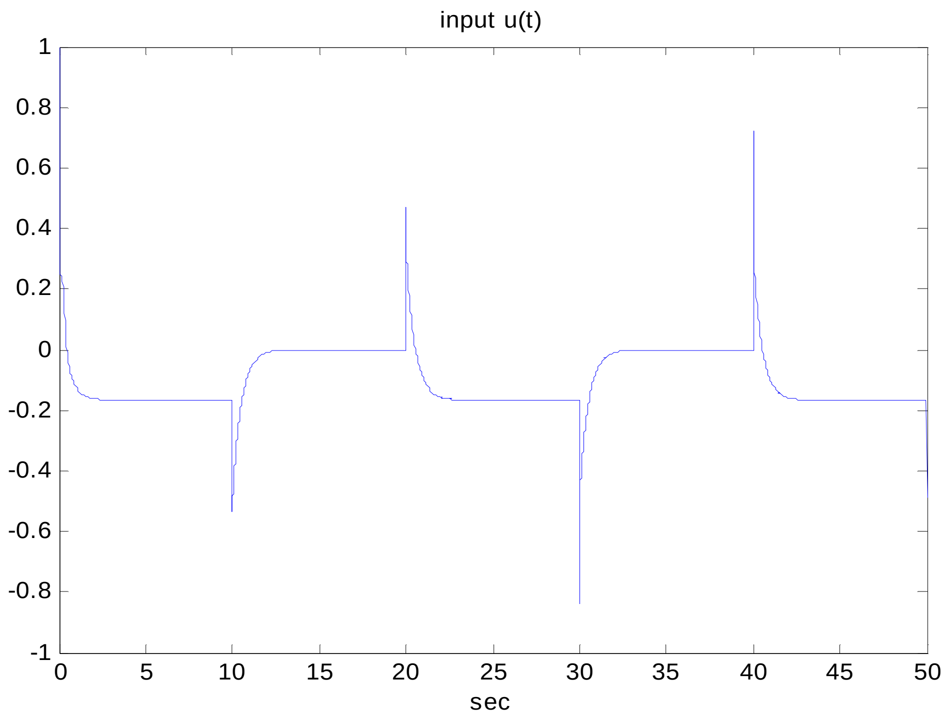

Figure 3.7. The Simulation Result of the Control Input.

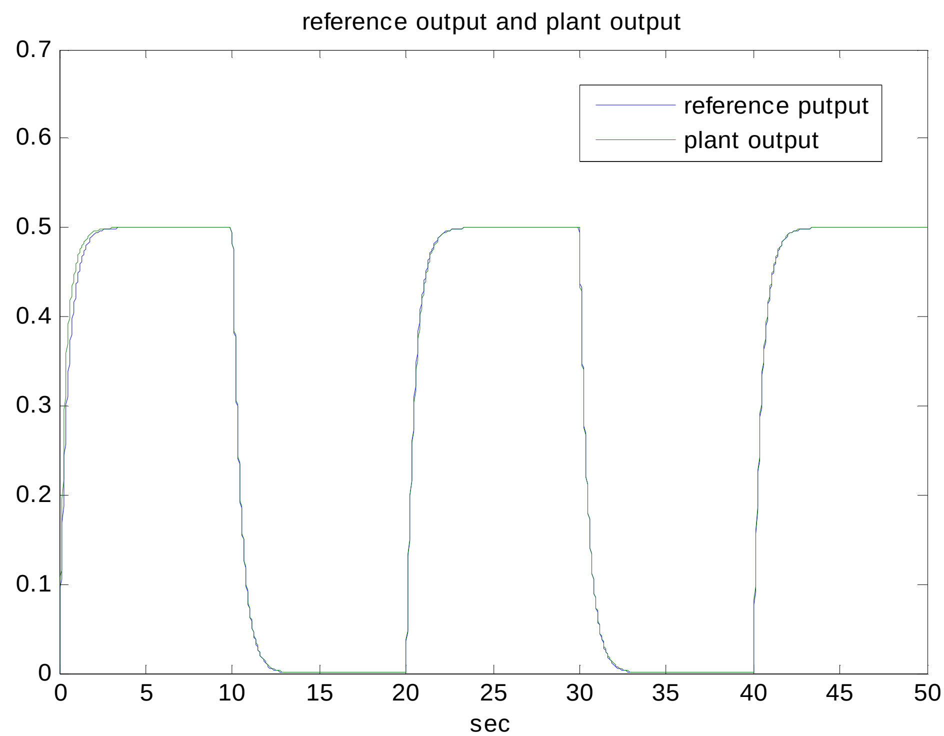

Figure 3.8. The Simulation Result of the Reference Output and Plant Output.

2. L1 Adaptive Control with Bounded Output Disturbance

Consider a system

$$y(t) = k_0 \frac{1}{p-1} u(t)$$

where $0.5 \le k_0 \le 3$, and the reference model is

$$y_m(t) = \frac{1}{p+1} v(t)$$

where

$$D(p) = p+1.$$

Then

$$B(p) = 1 \ , A(p) = p-1 \ , D(p) = p+1 \tag{3.115}$$

Assuming $k_0 = 2$ as the true value, the controller can be expressed as

$$0.5u(t) = -\begin{bmatrix} k_0 - k_m & h^T & k^T \end{bmatrix} \begin{bmatrix} u(t) \\ \underline{\phi}_u \\ \underline{\phi}_y \end{bmatrix} + v(t) = -\underline{\theta}^T \underline{\phi}(t) + v(t) \tag{3.116}$$

where

$$\begin{cases} \underline{\theta}^T = \begin{bmatrix} 0.5 & 2 & 6 \end{bmatrix}^T \\ \underline{\phi}(t)^T = \begin{bmatrix} u(t) & \dfrac{1}{p+5} u(t) & \dfrac{1}{p+5} y(t) \end{bmatrix}^T \end{cases} \tag{3.117}$$

In accordance with **Condition 1** and **Condition 2**, terms are chosen as $\omega_0 = 5$, $\omega_1 = 10$, $\omega_E = 100$, and $L = 2 \ge N - M = 1$. As in the previous example, Figure 3.9 and Figure 3.10 show that the stability conditions are satisfied.

With appropriate substitutions $C_F(p)$ and $C_E(p)$ can be expressed as:

$$C_F(p) = \left(\frac{\omega_0}{p+\omega_0}\right)\left(\frac{\omega_1{}^L}{\left(p+\omega_1\right)^L}\right) = \left(\frac{5}{p+5}\right)\left(\frac{10^2}{\left(p+10\right)^2}\right)$$

$$C_E(p) = \frac{\omega_E{}^L}{(p+\omega_E)^L} = \frac{100}{(p+100)^2}$$

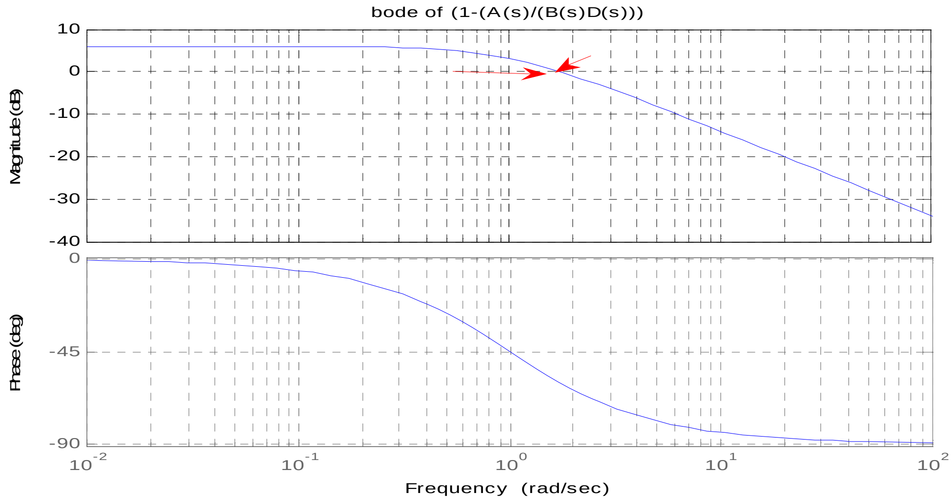

Figure 3.9. The Bode Plot of $\left(1-\dfrac{A(p)}{B(p)D(p)}\right)$.

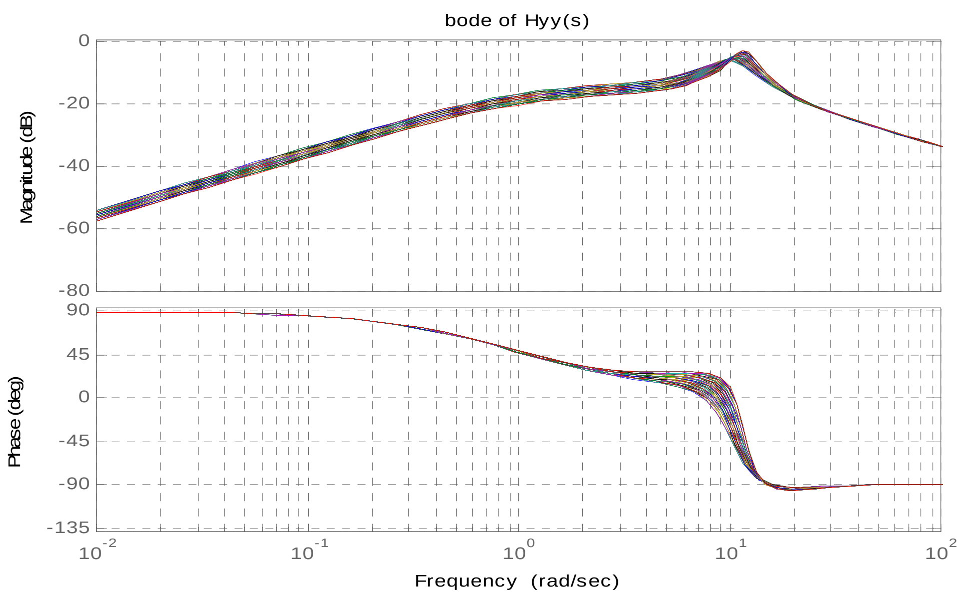

Figure 3.10. The Bode Plot of $H_{yy}(p)$ for 100 Plant Realizations.

Figure 3.11 and Figure 3.12 show the results of this simulation.

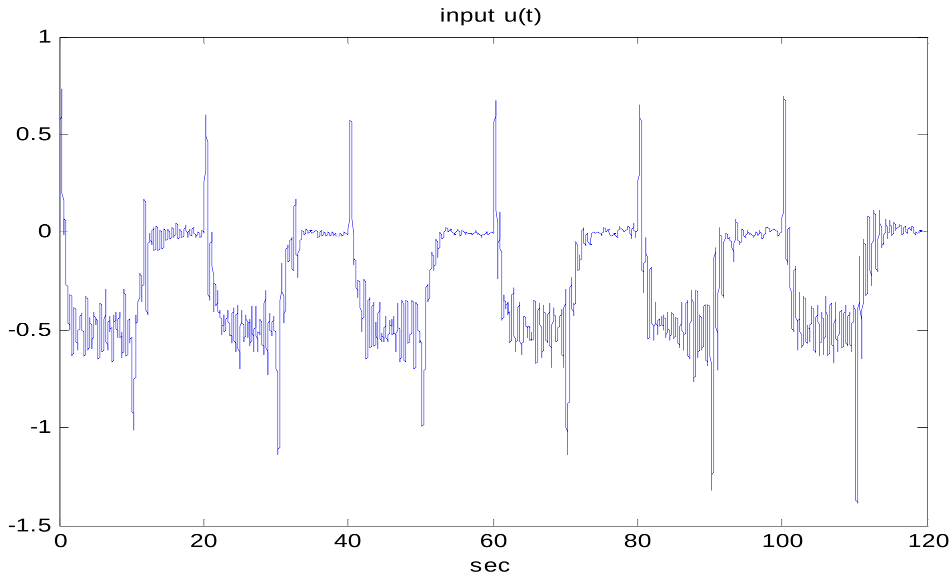

Figure 3.11. The Simulation Result of the Control Input.

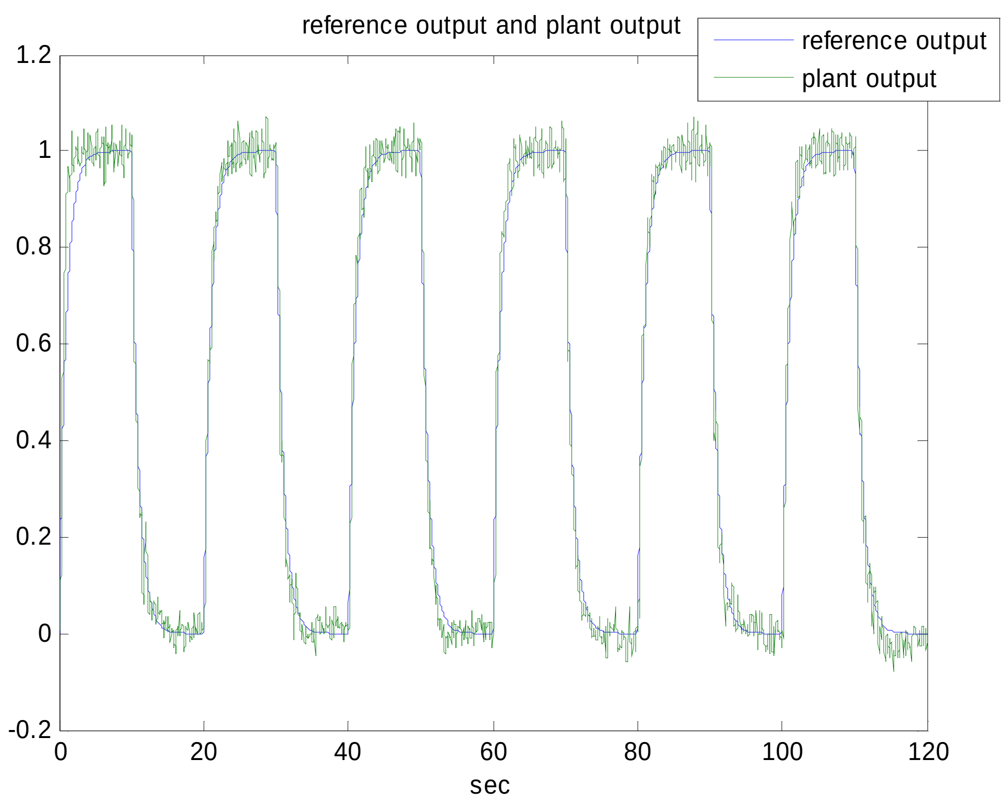

Figure 3.12 The Simulation Result of the Reference Output and Plant Output.

3. Example with Sensor Noise

Consider a plant with transfer function

$$k_0 \frac{B(p)}{A(p)} = k_0 \frac{1}{p+a} \tag{3.118}$$

where the parameters are within known bounds

$$-2 \le a \le 0\,,\ 0.5 \le k_0 \le 3 \tag{3.119}$$

Let the reference model be

$$\frac{1}{D(p)} = \frac{1}{p+1}. \tag{3.120}$$

First, the LPF, $C_F(p)$ is chosen and is of the form

$$C_F(p) = \left(\frac{\omega_0}{p+\omega_0}\right)\left(\frac{\omega_1^{\,L}}{\left(p+\omega_1\right)^L}\right) = C_0(p)C_1(p). \tag{3.121}$$

Again, we choose $L = 2 \ge N - M = 1$. From the bound on k_0, we obtain $0 \le \gamma \le \gamma_{MAX} = \frac{k_M}{k_m} - 1 = \frac{3}{0.5} - 1 = 5$ and the Bode plot of $\left(1 - \frac{A(p)}{B(p)D(p)}\right)$ is shown in Figure 3.13 for a number of random realizations of the plant.

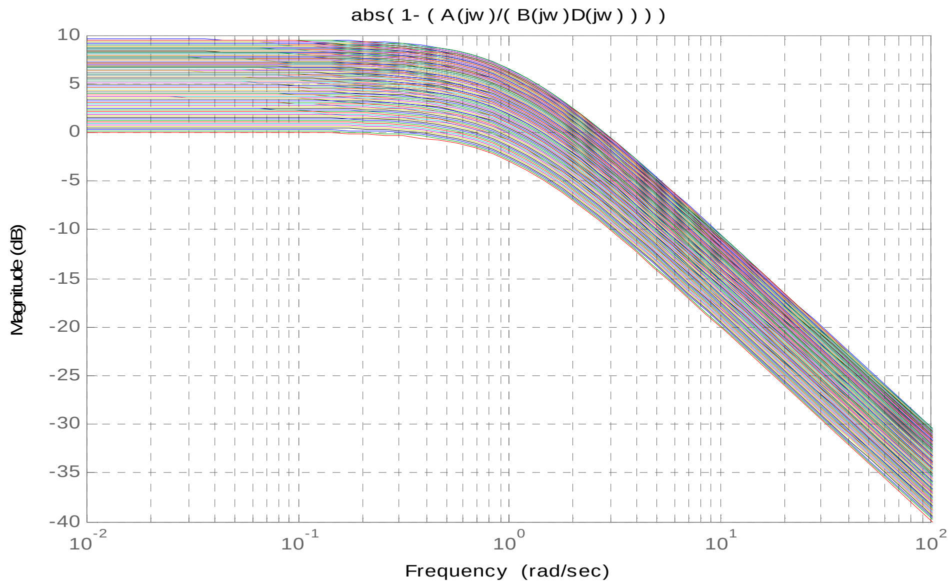

Figure 3.13. Frequency Response of $\left(1-\dfrac{A(p)}{B(p)D(p)}\right)$ for 100 Plant Realizations.

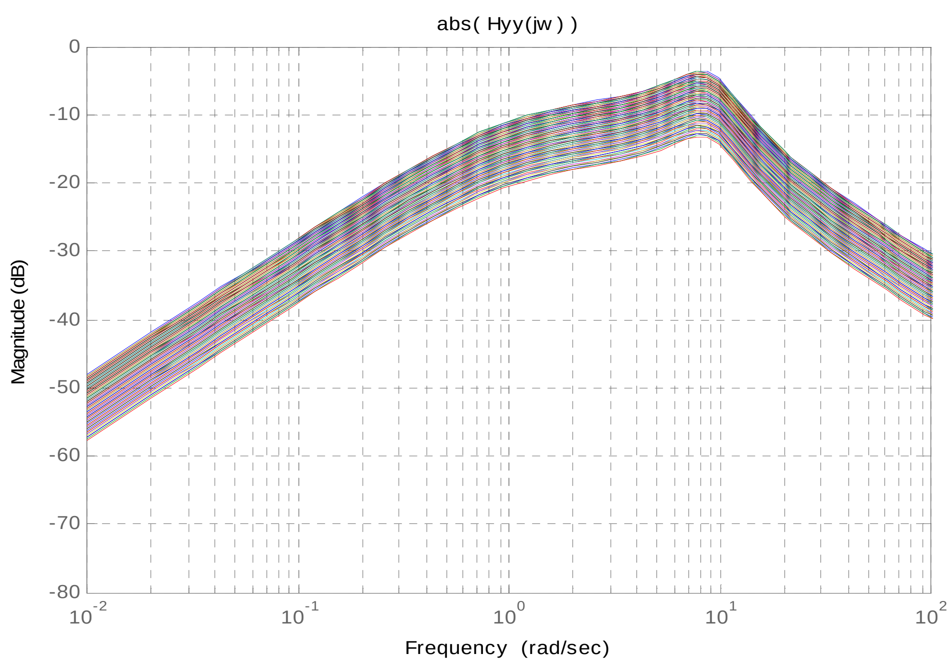

Figure 3.14. Frequency Response of $H_{yy}(p)$ for 100 Plant Realizations.

It can easily be seen that for $\omega > 3$ rad/sec all realizations have magnitude smaller than one that required by **Condition 1**. In this example, $\omega_0 = 5$ rad/sec is chosen so that **Condition 1** holds for all realizations, and $\omega_1 = 10$ rad/sec so that the gain margin of $C_F(p)$ is larger than $20\log_{10}\gamma_{MAX} = 14dB$. This idea can be applied to the controller design.

From Figure 3.14, the magnitudes $\left|H_{yy}(p)\right|$ of all realizations are found to be smaller than one required to satisfy **Condition 2**. Both of these results guarantee the stability of $H_{ref}(p)$ in Equation (3.40) for all realizations of the plant.

The responses of the inputs and output signals for this adaptive control system for two different values of ω_E are shown in Figure 3.15 and Figure 3.16. To ensure a more realistic experiment, bandlimited random observation noise is added. The simulation results are shown in Figures 3.17 and Figure 3.18. Notice that, as predicted by the theoretical arguments in the previous section, the peak of the transient response decreases as the bandwidth ω_E increases. The relationship between ω_0, ω_E, and the transient performance is evident in Equation (3.87).

$$\left|e_\phi(t)\right| \le k_L \left(\frac{\omega_0}{\omega_E}\right)^{\frac{1}{4}} \sup_{0\le\tau\le t}\left\|\phi(\tau)\right\| \tag{3.122}$$

In the next section, the upper bounds of parameters and transfer functions is discussed.

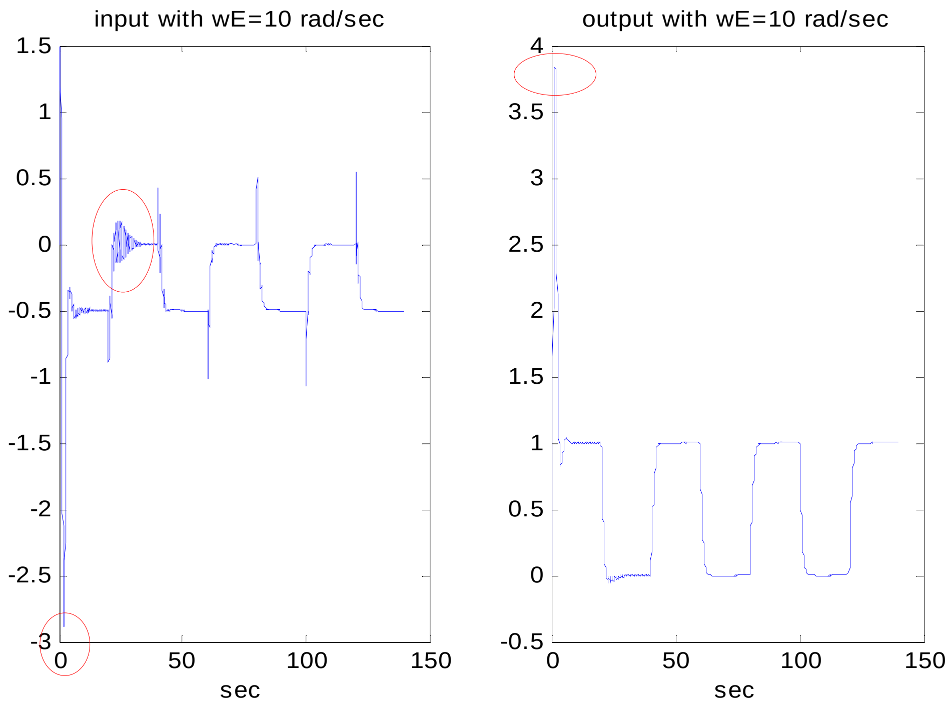

Figure 3.15. Frequency Response to Square Wave for $\omega_E = 10$ rad/sec.

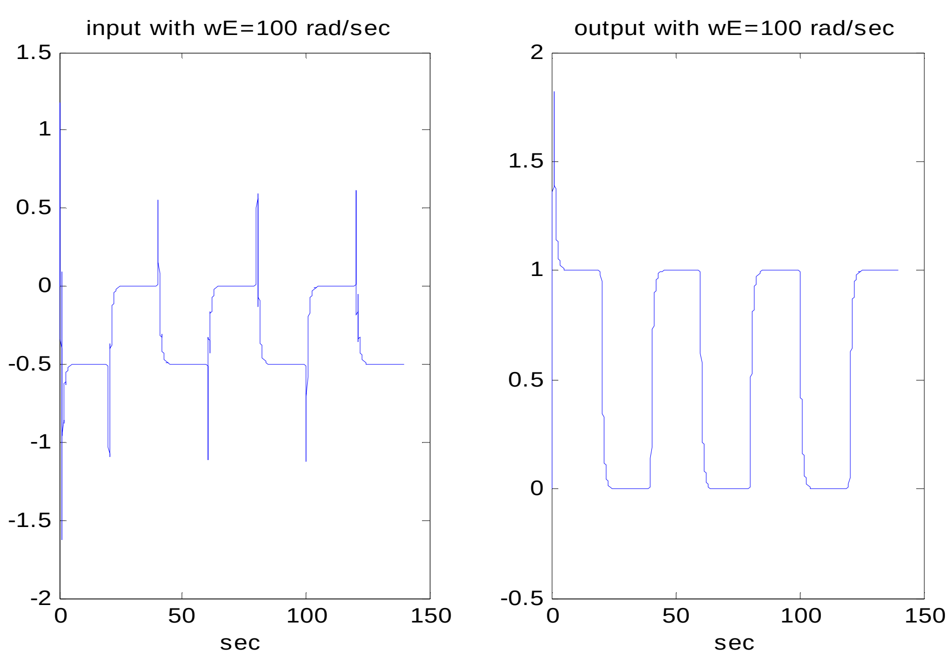

Figure 3.16 Frequency Response to Square Wave for $\omega_E = 100$ rad/sec.

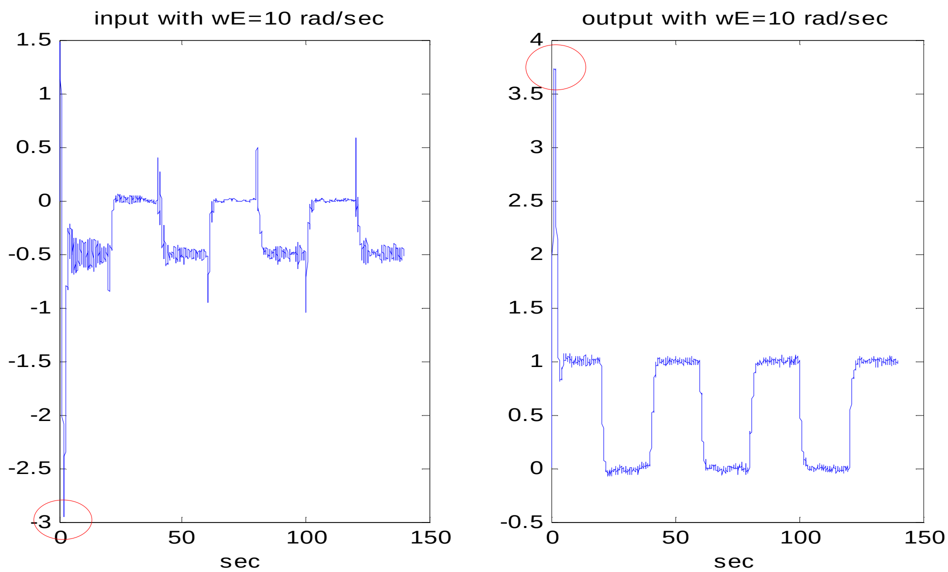

Figure 3.17. Frequency Response to Square Wave for $\omega_E = 10$ rad/sec with Bandlimited Random Observation Noise.

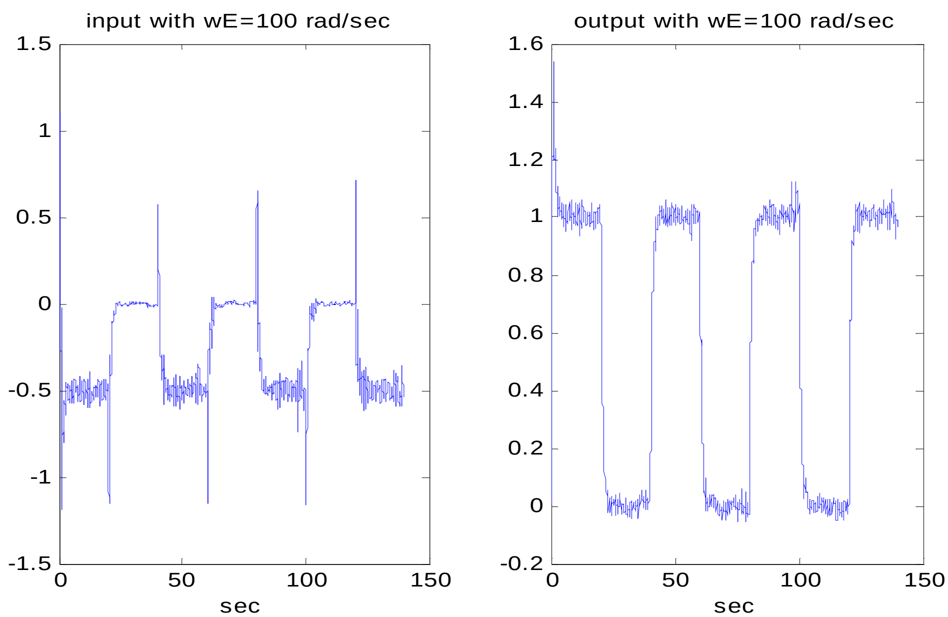

Figure 3.18. Frequency Response to Square Wave for $\omega_E = 100$ rad/sec with Bandlimited Random Observation Noise.

4. Example with Modeling Error

In this case, the adaptive controller presented in the previous section is used, but the plant has unmodeled dynamics as represented by

$$y(t) = k_0 \frac{1}{p-a} \frac{200}{p^2 + 30p + 200} u(t) \tag{3.123}$$

In other words, the system is treated as a first order system, while in reality it has two extra poles affecting the high frequency only. It is well known that in this situation the adaptive controller can easily become unstable and sensitive to external disturbances. However, the proposed controller seems to retain stability and tracking properties. Simulation results are shown in Figure 3.19 and Figure 3.20.

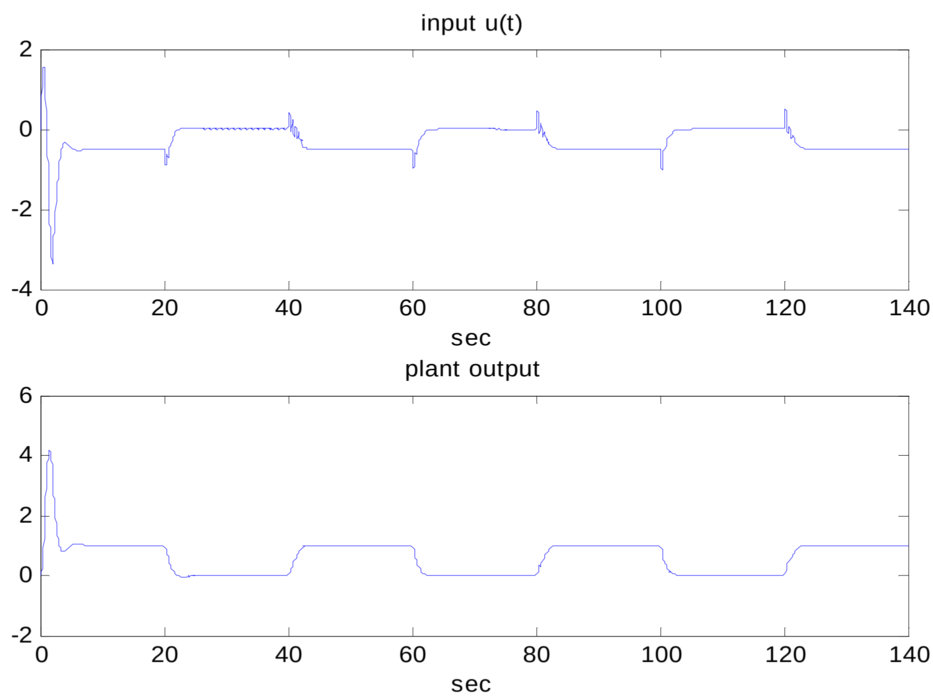

Figure 3.19. Simulations with Unmodeled Dynamics $\dfrac{200}{p^2 + 30p + 200}$.

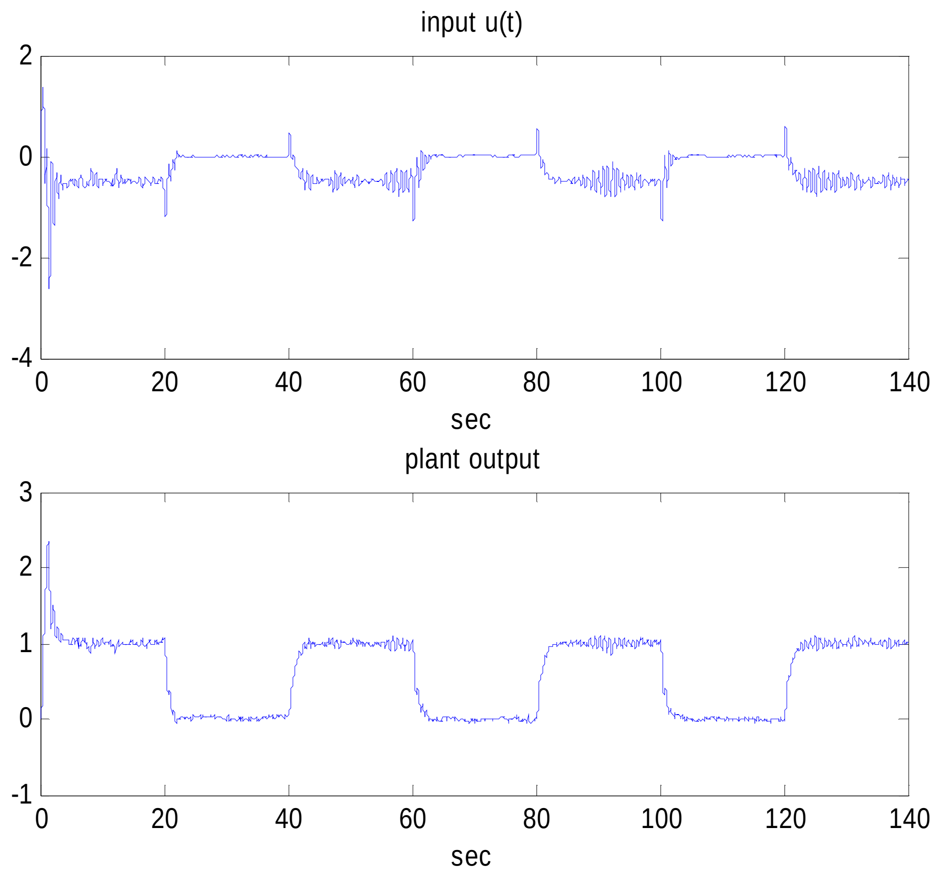

Figure 3.20. Simulations with Unmodeled Dynamics $\dfrac{200}{p^2+30p+200}$ and Sensor Noise.

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IV. L1 ADAPTIVE CONTROL FOR NON-MINIMUM PHASE SYSTEMS

A. MOTIVATION AND BACKGROUND

The direct Adaptive Control technique presented in the previous chapters assumes that the system to be controlled is minimum phase, in the sense that the zeros of its transfer function have to be on the left half side of the s-plane.

It is well known that non-minimum phase zeros present very important limitations on the performance of control systems. Most important is the fact that, while poles can be moved by state feedback, zeros can be replaced only by pole-zero cancellation. Since unstable pole-zero cancellation leads to unstable, uncontrollable, or unobservable modes, this operation is clearly not feasible.

Non-minimum phase systems are very important in some applications and therefore cannot be ignored. In particular, systems with flexible appendages most likely have zeros on the imaginary axis and are therefore non-minimum phase [89].

In this chapter, the adaptive control of this class of systems is discussed. The non-minimum phase limitation is avoided by controlling an augmented plant obtained by the addition of a system in parallel to the plant to be controlled. This augmenting network has to be designed such that its output is zero in steady state, and the overall combination is minimum phase.

Although the control presented for minimum phase systems is restrictive, it does exhibit desired tracking in the presence of wide plant uncertainties. Several computer simulations show the effectiveness of such a controller. Furthermore, implementation on an experimental flexible mechanical arm, with uncertain dynamics, shows that this approach can be a viable technique for the control of this class of systems.

B. FUNDAMENTAL PRINCIPLE AND ANALYSIS OF THE MODIFIED L1 ADAPTIVE CONTROL

Along similar lines as in the previous chapters, consider a Single Input Single Output (SISO) system with dynamics

$$y(t) = k_0 \frac{B(p)}{A(p)} u(t). \tag{4.1}$$

with possible non minimum phase zeros. When this is the case, as shown in Figure 4.1, we augment it by a parallel system so that the combination system becomes

$$\bar{y}(t) = \left(\frac{k_0 B(p)}{A(p)} + \frac{E_1(p)}{D_1(p)} \right) u(t) = \frac{A(p)E_1(p) + k_0 B(p) D_1(p)}{A(p) D_1(p)} u(t). \tag{4.2}$$

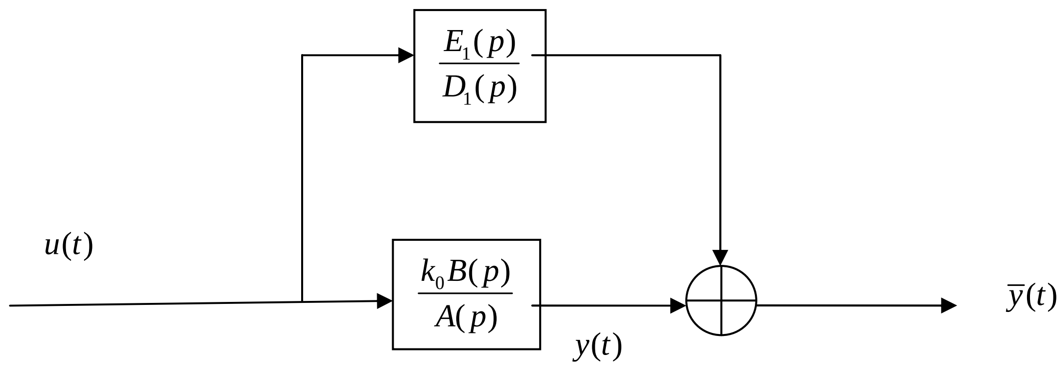

Figure 4.1. New Closed Loop Dynamics with a Designed PID $\frac{E_1(p)}{D_1(p)}$.

Now, the problem is to choose the augmenting transfer function $\frac{E_1(p)}{D_1(p)}$ to satisfy the following requirements:

a.) the numerator of the combined system $\left(A(p)E_1(p) + k_0 B(p) D_1(p)\right)$ is Hurwitz;

b.) in steady state the output of the augmenting system goes to zero.

To satisfy Requirement b.) when the input is a constant, $E_1(p)$ is expressed as

$$E_1(p) = p \bullet E_1^{/}(p) \tag{4.3}$$

so that the derivative action (the term " p ") makes the steady state value zero when the input is a constant. To satisfy Requirement a.), notice that the numerator of Equation (4.2) is the characteristic polynomial of the system shown in Figure 4.2.

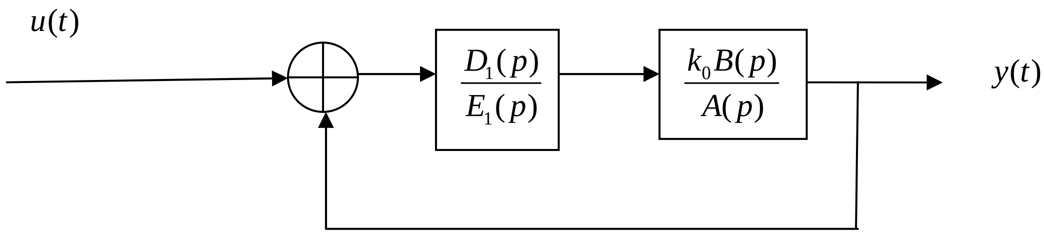

Figure 4.2. The Other Expression of the New Closed Loop Dynamics with a Designed PID.

Therefore, the augmenting network $\frac{E_1(p)}{D_1(p)}$ must be such that its inverse $\frac{D_1(p)}{E_1(p)}$ yields a stable closed loop system. Furthermore, since it is desired that $\frac{E_1(p)}{D_1(p)}$ is strictly proper, $\frac{D_1(p)}{E_1(p)}$ will not be proper.

A possible choice is that $\frac{D_1(p)}{E_1(p)}$ is a PID controller, with the integral action to satisfy Equation (4.3), and the derivative action to make its inverse proper. For example, let a nominal plant be expressed as

$$k_0 \frac{B(p)}{A(p)} = k_0 \frac{p^2+1}{p^4+2p^2} = k_0 \frac{(p+1j)(p-1j)}{p^2(p+1.4142j)(p-1.4142j)} \tag{4.4}$$

Observe that there are two zeros located on the imaginary axis and therefore, it is non-minimum phase.

In order to make the system minimum phase, a PID compensator $\frac{D_1(p)}{E_1(p)}$ must be designed so that the system in Figure 4.2 is stable. The augmented minimum phase system is shown in Equation (4.2).

From Figure 4.1, the new output is shown to be

$$\bar{y}(t) = y(t) + \frac{E_1(p)}{D_1(p)}u(t) = \frac{A(p)E_1(p) + B(p)D_1(p)}{A(p)D_1(p)}u(t). \tag{4.5}$$

Based on the requirements listed above, the degree of $E_1(p)$ has to be less than $D_1(p)$. Further, according to the final value theorem [89–90], the numerator $E_1(p)$ must have roots at its “zeros“. Thus define

$$E_1(p) = p \bullet E_1^{/}(p) \tag{4.6}$$

$$D_1(p) = a_0 p^D + a_1 p^{D-1} + \cdots + a_D \tag{4.7}$$

and choose

$$E_1(p) = p \tag{4.8}$$

$$D_1(p) = p^2 + p + 0.1 \tag{4.9}$$

Then the transfer function of the augmented system becomes

$$\frac{BB(p)}{AA(p)} = \frac{A(p)E_1(p) + B(p)D_1(p)}{A(p)D_1(p)} = \frac{p^5 + p^4 + 3p^3 + 1.1p^2 + p + 0.1}{p^6 + p^5 + 2.1p^4 + 2p^3 + 0.2p^2} \tag{4.10}$$

with zeros at

$$p = \begin{bmatrix} -0.3347 + 1.5021i & -0.3347 - 1.5021i & -0.1107 + 0.6114i & -0.1107 - 0.6114i & -0.1094 \end{bmatrix} \tag{4.11}$$

Which are strictly located in the LHS s-plane. This leads to the augmented system

$$\bar{y}(t) = k_0 \frac{BB(p)}{AA(p)} u(t) = \frac{p^5 + p^4 + 3p^3 + 1.1p^2 + p + 0.1}{p^6 + p^5 + 2.1p^4 + 2p^3 + 0.2p^2} u(t). \qquad (4.12)$$

Now, the reference model is chosen as in Equation (3.4):

$$y_m(t) = \frac{1}{p+1} v(t) \qquad (4.13)$$

where $D(p) = p+1$ is a Hurwitz polynomial representing the desired closed loop dynamics. The degree of $D(p)$ is $N - M$, which is equal to the relative degree of the augmented plant, and $v(t)$ is an arbitrary external bounded input.

Designing the standard adaptive controller for the augmented minimum phase system follows. In particular, let the filter be chosen as

$$C_F(p) = \left(\frac{\omega_0}{p+\omega_0}\right)\left(\frac{\omega_1^2}{\left(p+\omega_1\right)^2}\right) = \left(\frac{10}{p+10}\right)\left(\frac{10^2}{\left(p+10\right)^2}\right) \qquad (4.14)$$

It is possible to verify that this guarantees stability of the reference model,

$$\max_{\omega} \max_{0 \le \gamma \le \gamma_{MAX}} \left| H_{yy}\left(j\omega\right) \right| < 1 \qquad (4.15)$$

where $H_{yy}\left(p\right)$ is as defined in Chapter III. Simulation results shown in Figures 4.3–4.5 show the stability and asymptotic tracking of the desired trajectory.

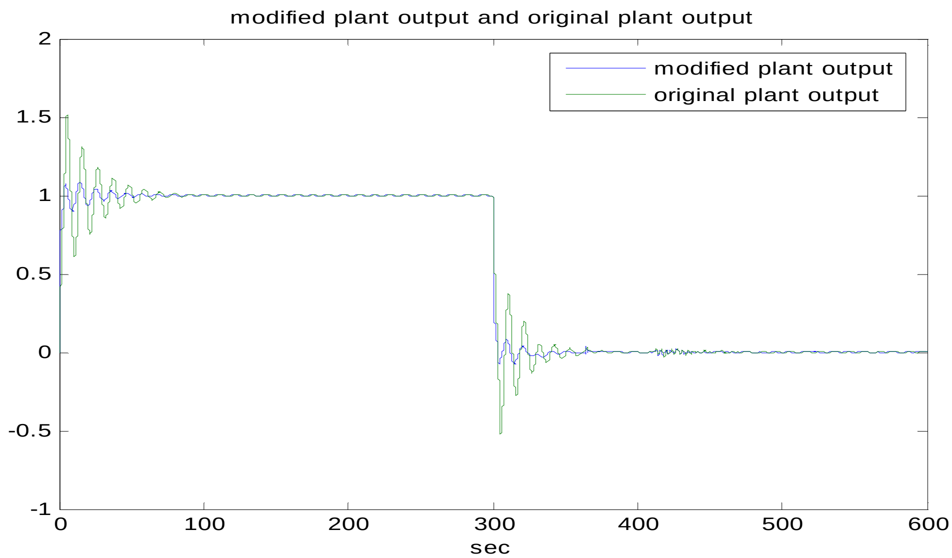

Figure 4.3. Modified Plant Output and Original Plant Output.

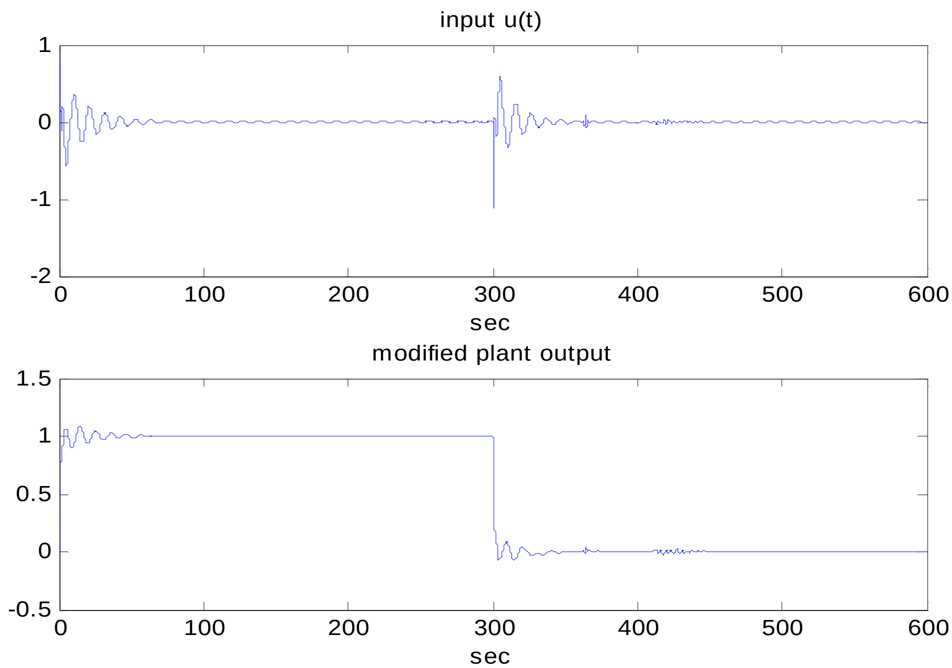

Figure 4.4. Simulation Result of Modified L1 Adaptive Control System.

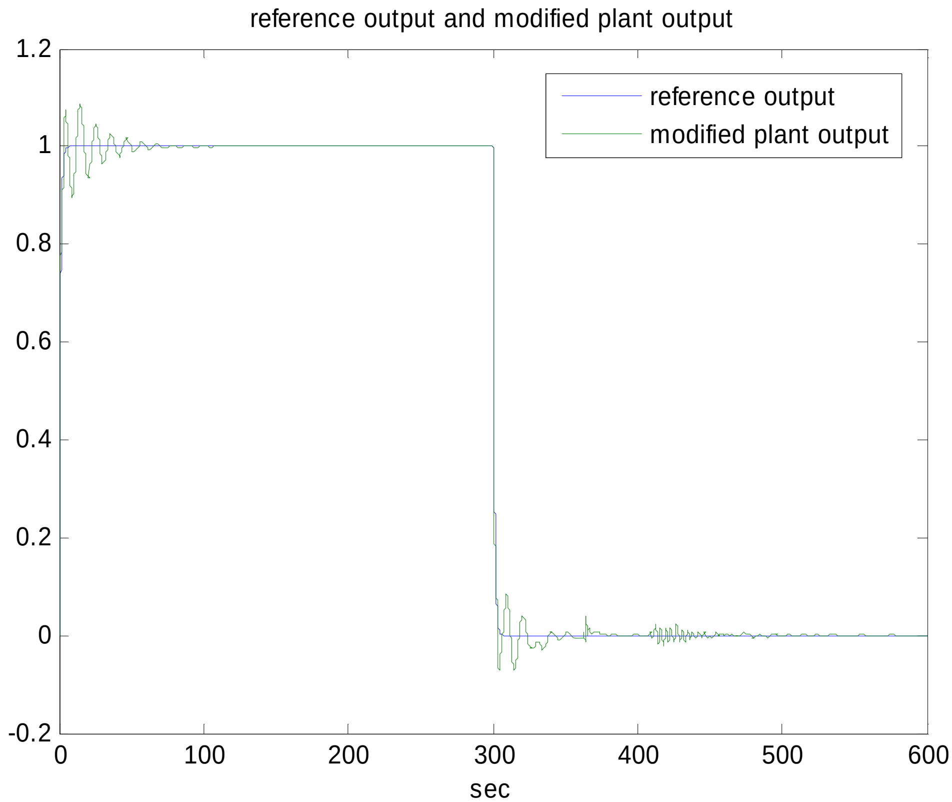

Figure 4.5. Modified Plant Output Tracks the Reference Output.

C. SIMULATIONS OF MODIFIED L1 ADAPTIVE CONTROL WITH NON-MINIMUM PHASE SYSTEM

In order to verify the robustness and stability of the modified L1 adaptive controller with PID element, a modified controller is adapted to perform under realistic conditions.

1. Modified L1 Adaptive Control System with Bounded Output Disturbance

Consider a system

$$y(t) = k_0 \frac{B(p)}{A(p)} u(t) = \frac{0.06023p^2 + 0.13378}{p^4 + 2.57184p^2} u(t) \tag{4.16}$$

where $0.5 \le k_0 \le 3$, assuming $k_0 = 1$ and the reference output

$$y_m(t) = \frac{1}{p+1} v(t).$$

Then

$$B(p) = 0.06023 p^2 + 0.13378, \; A(p) = p^4 + 2.57184 p^2, \; D(s) = p+1 \tag{4.17}$$

This plant has two zeros, one located at (0 + 1.4904i) and the other at (0 - 1.4904i), both located on the imaginary axis, and therefore the system is non-minimum phase.

Next, a proper PID control $\frac{E_1(p)}{D_1(p)}$ is defined with

$$E_1(p) = p \tag{4.18}$$

and

$$D_1(p) = 10 p^2 + 5p + 0.1 \tag{4.19}$$

It is easy to verify that this controller $\frac{E_1(p)}{D_1(p)}$ stabilizes the system. After the system is augmented with $\frac{E_1(p)}{D_1(p)}$, the response is

$$\bar{y}(t) = k_0 \frac{BB(p)}{AA(p)} u(t) = \frac{p^5 + 0.6023 p^4 + 2.8730 p^3 + 1.3438 p^2 + 0.6689 p + 0.0134}{p^6 + 0.5 p^5 + 2.5818 p^4 + 1.2859 p^3 + 0.0257 p^2} u(t). \tag{4.20}$$

Now, the reference model is chosen as

$$y_m(t) = \frac{1}{p+5} v(t) \tag{4.21}$$

where $D(p) = (p+5)$ is a Hurwitz polynomial representing the desired closed loop dynamics. The degree of $D(p)$ is $N - M$, which equals the relative degree of the new plant, and $v(t)$ is an arbitrary external bounded input.

Furthermore, in order to satisfy the stability conditions of Chapter III, $\omega_0 = 30$ rad/sec, $\omega_1 = 30$ rad/sec, and $\omega_E = 50$ rad/sec are chosen. Then the LPF becomes

$$C_F(p) = \left(\frac{\omega_0}{p+\omega_0}\right)\left(\frac{\omega_1^{\ 2}}{(p+\omega_1)^2}\right) = \left(\frac{30}{p+30}\right)\left(\frac{30^2}{(p+30)^2}\right) \tag{4.22}$$

and

$$C_E(p) = \frac{\omega_E^{\ L}}{(p+\omega_E)^L} = \frac{50^2}{(p+50)^2}. \tag{4.23}$$

We can verify that, with this LPF,

$$\max_{\omega} \max_{0 \le \gamma \le \gamma_{MAX}} \left| H_{yy}(j\omega) \right| < 1 \tag{4.24}$$

and the transfer functions : $H_{yy}(p)$, $H_{y0}(p)$, $H_{u0}(p)$, and $H_{uy}(p)$ can be obtained from Equation (3.42) as before.

Figures 4.6–4.9 show the results of this simulation. From Figure 4.6, it can be seen that the augmented plant is stabilized. In the presence of a bounded output disturbance, the settling time is a little longer compared to the example with no disturbance in last section. However, due to the effect of the additional PID element, the original output tracks the modified plant output and it is still stable, despite the output disturbance. In addition, from Figures 4.7 and 4.8, it is obvious that this modified L1 adaptive control system satisfies the specified requirements of this experiment. It is evident that there is no large control action or poor transient performance during the duration of execution, and the modified plant output did track the reference output. Figure 4.9, shows that the adaptive error is small and bounded, which satisfies the requirement of a stable adaptive control system.

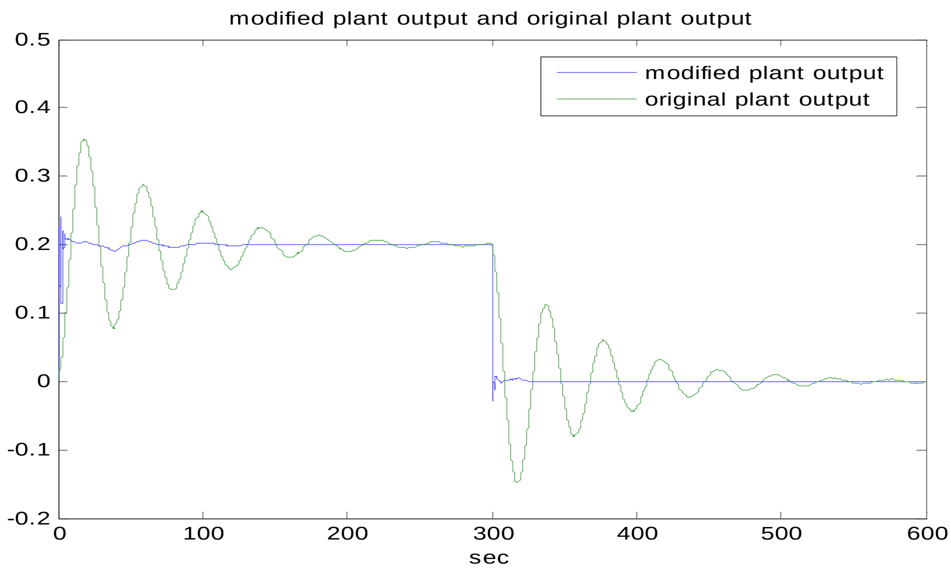

Figure 4.6. Modified Plant Output and Original Plant Output with Bounded Output Disturbance.

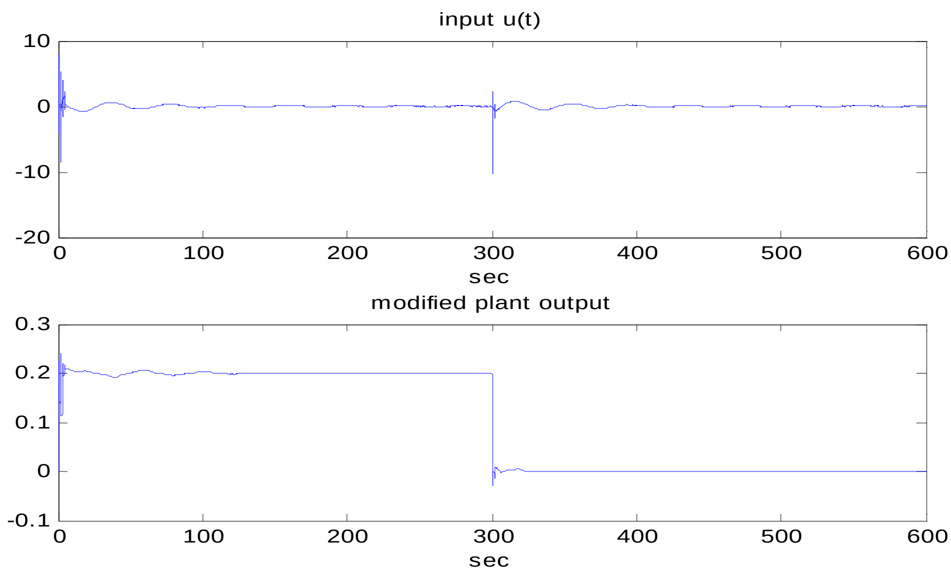

Figure 4.7. Simulation Result of Modified L1 Adaptive Control System with Bounded Output Disturbance.

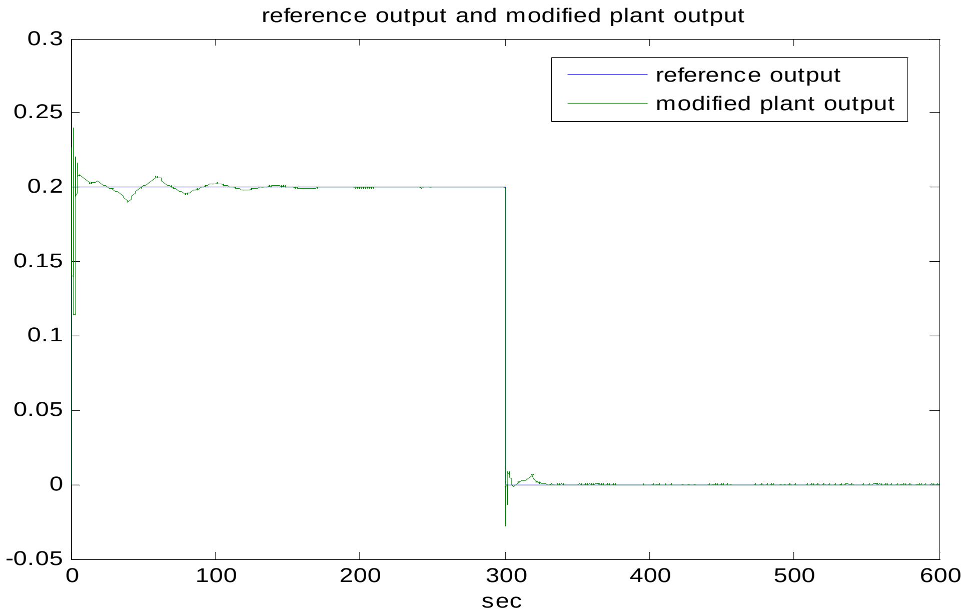

Figure 4.8. Modified Plant Output Tracks the Reference Output with Bounded Output Disturbance.

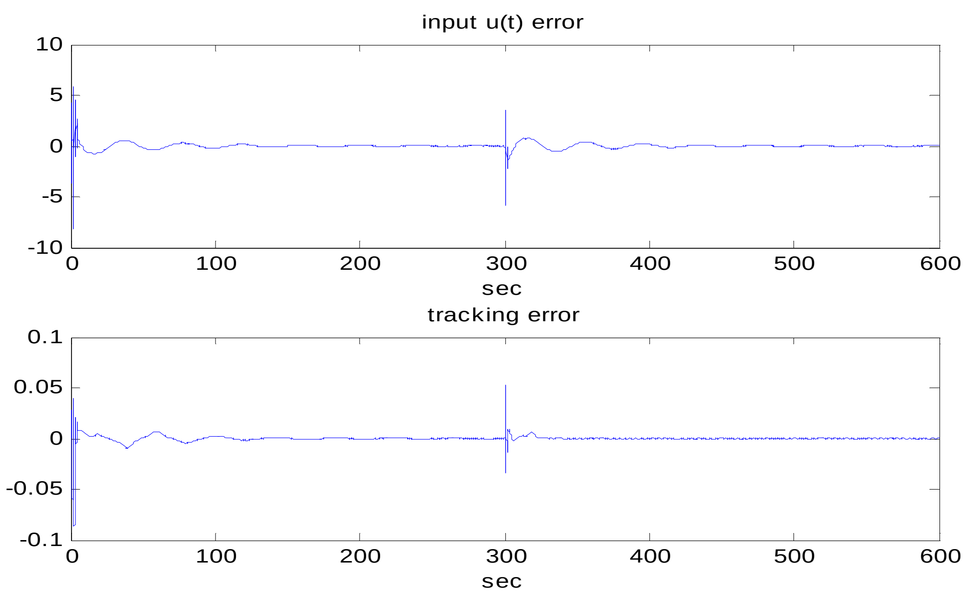

Figure 4.9. The Adaptive Error of Modified Plant Output with Bounded Output Disturbance.

2. Modified L1 Adaptive Control System in the Presence of Unmodeled Dynamics

In this case, the same adaptive controller is used as in the previous example, but the plant has unmodeled dynamics as represented by

$$y(t) = k_0 \frac{0.06023p^2 + 0.13378}{p^4 + 2.57184p^2} \frac{300}{p^2 + 10p + 300} u(t) \tag{4.25}$$

where $k_0 = 0.1$, $k_m = 0.9 * k_0 = 0.09 \le k_0 \le 1.1 * k_0 = k_M = 0.11$ is assumed. In addition, from Equation (4.25), it has two extra poles affecting the high frequencies only. In general, in this situation, the adaptive controller can easily become unstable and sensitive to external disturbances. Also, the plant has two zeros, one located at (0 + 1.4904i) and the other at (0 - 1.4904i), both on the imaginary axis and therefore the system is non-minimum phase.

Next, the proper control is defined as $\frac{E_1(p)}{D_1(p)}$, the inverse of a PID control, where

$$E_1(p) = p \tag{4.26}$$

and

$$D_1(p) = 10p^2 + 5p + 0.1. \tag{4.27}$$

It is easy to verify that this controller $\frac{E_1(p)}{D_1(p)}$ stabilizes the system. After augmenting the system with $\frac{E_1(p)}{D_1(p)}$, the response is

$$\overline{y}(t) = k_0 \frac{BB(p)}{AA(p)} u(t) = k_0 \frac{p^5 + 0.6023p^4 + 2.8730p^3 + 1.3438p^2 + 0.6689p + 0.0134}{p^6 + 0.5p^5 + 2.5818p^4 + 1.2859p^3 + 0.0257p^2} u(t). \tag{4.28}$$

Now, the reference model is chosen as

$$y_m(t) = \frac{1}{p+5} v(t) \tag{4.29}$$

with

$$D(p) = p+5 \tag{4.30}$$

where $D(p) = (p+5)$ is a Hurwitz polynomial representing the desired closed loop dynamics. The degree of $D(p)$ is $N-M$, which equals the relative degree of the new plant, and $v(t)$ is an arbitrary external bounded input.

Furthermore, in order to satisfy the stability conditions of Chapter III, $\omega_0 = 30$ rad/sec, $\omega_1 = 30$ rad/sec, and $\omega_E = 50$ rad/sec are chosen. Then the LPF becomes

$$C_F(p) = \left(\frac{\omega_0}{p+\omega_0}\right)\left(\frac{\omega_1^{\,2}}{(p+\omega_1)^2}\right) = \left(\frac{30}{p+30}\right)\left(\frac{30^2}{(p+30)^2}\right) \tag{4.31}$$

and

$$C_E(p) = \frac{\omega_E^{\,L}}{(p+\omega_E)^L} = \frac{50^2}{(p+50)^2} \tag{4.32}$$

It is easy to verify that, with this LPF,

$$\max_{\omega} \max_{0 \le \gamma \le \gamma_{MAX}} \left| H_{yy}(j\omega) \right| < 1 \tag{4.33}$$

and the transfer functions : $H_{yy}(p)$, $H_{y0}(p)$, $H_{u0}(p)$, and $H_{uy}(p)$ can be easily obtained from Equation (3.42).

Figures 4.10–4.12 show the simulation results of this experiment. From Figure 4.10 and Figure 4.11 it is obvious that this modified L1 adaptive control system satisfies

the specified requirement. It is evident that there is no large control action or poor transient performance during the duration of execution, and the modified plant output did track the reference output. In Figure 4.12, the adaptive error is shown and is small and bounded, which satisfies the requirement of a stable adaptive control system.

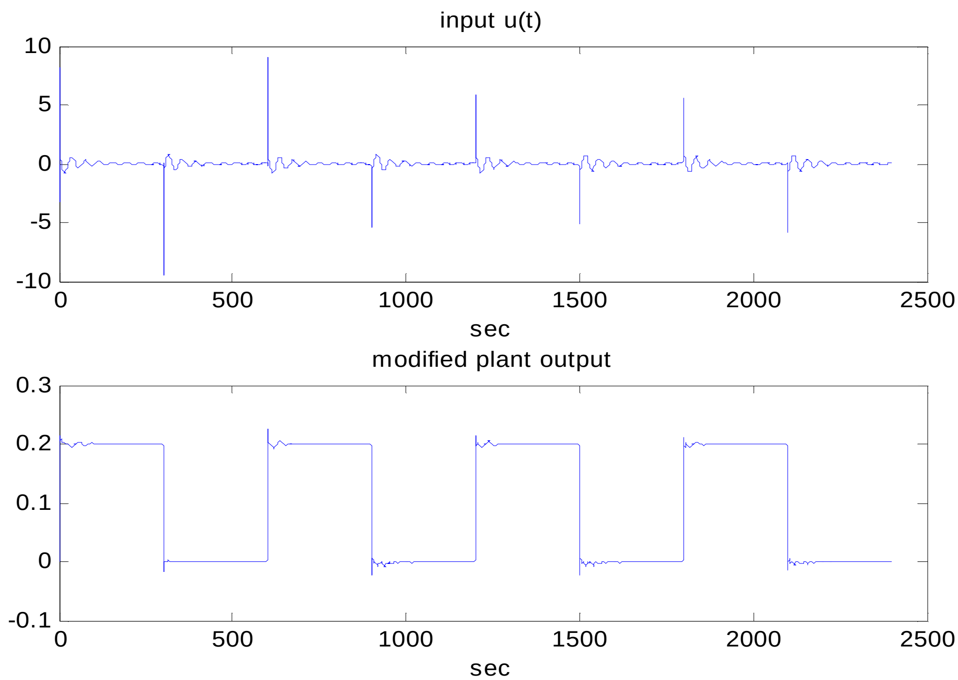

Figure 4.10. Simulation Result of Modified L1 Adaptive Control System with Unmodeled Dynamics $\left(\dfrac{300}{p^2+10p+300}\right)$.

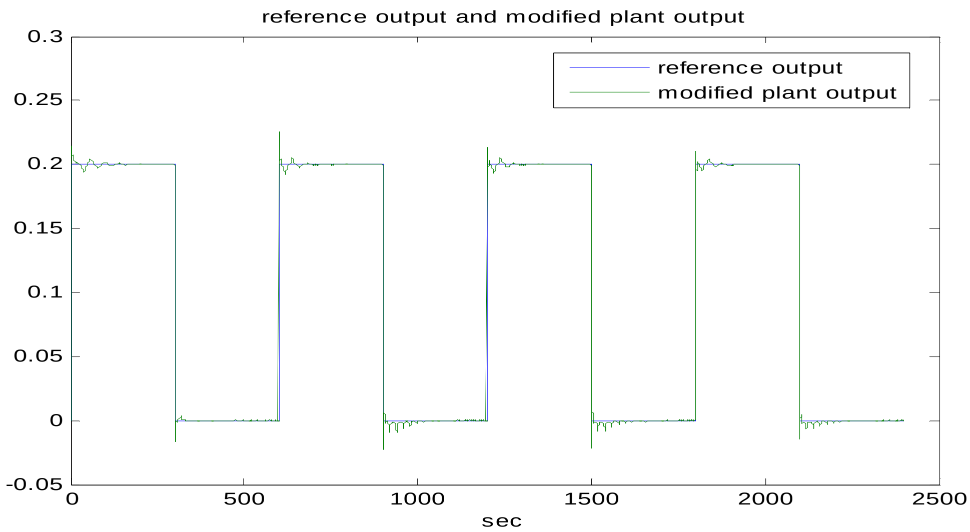

Figure 4.11. Modified Plant Output Tracks the Reference Output with Unmodeled Dynamics $\left(\dfrac{300}{p^2+10p+300}\right)$.

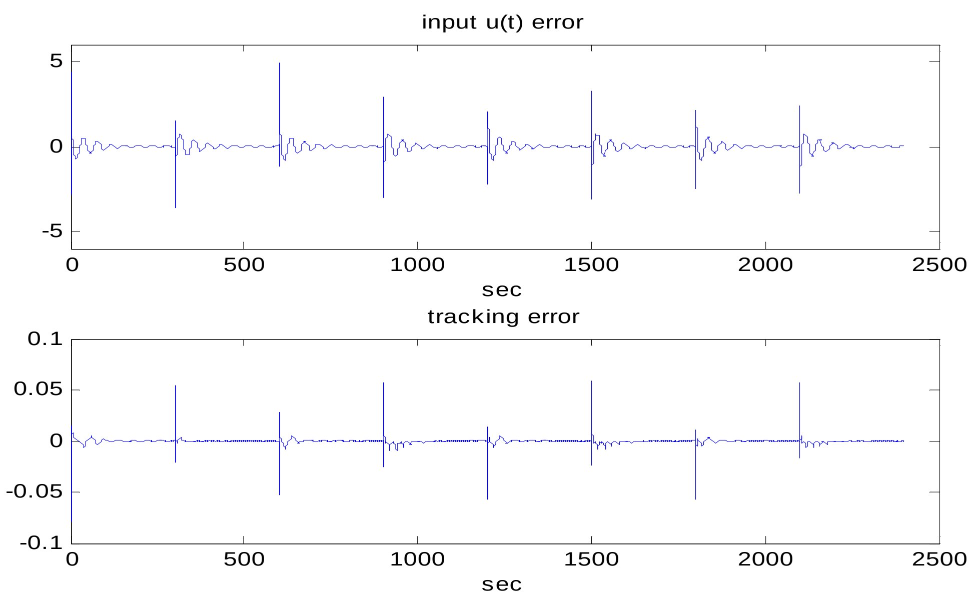

Figure 4.12. The Adaptive Error of a Modified Plant Output with Unmodeled Dynamics $\left(\dfrac{300}{p^2+10p+300}\right)$.

D. APPLICATION TO A SATELLITEWITH A FLEXIBLE ARM

In this section, the problem of applying the proposed adaptive controller to the control of a flexible arm is addressed. This is part of the equipment available in the Satellite Research and Design Center (SRDC) at the Naval Postgraduate School. The SRDC aims to recreate realistic space conditions in a laboratory environment.

The particular equipment used in this experiment is the Flexible Spacecraft Simulator (FSS) shown in Figure 4.13. The FSS simulates a flexible arm moving in the plane.

Figure 4.13. Flexible Spacecraft Simulator (FSS).

It was developed to experimentally verify the effectiveness, robustness and stability of designed control laws. It is comprised of a rigid central body and a flexible appendage. The rigid central body represents the main body of the spacecraft, and the flexible appendage represents a flexible antenna support structure [91]. In addition, the flexible appendage is composed of a based beam cantilevered to the main body and tip beam rigidly connected to the base beam at a right angle. Five air pads on a granite table support the FSS and its flexible appendage to minimize friction during motion.

The flexible model is similar to the mass-spring-damping model. Based on experimental, real data from FSS, the equations of motion of the flexible model can be developed and are written as

$$M\{\ddot{x}\} + K\{x\} = B_1 u \tag{4.34}$$

and

$$M = \begin{bmatrix} 19.2253 & -1.6190 \\ -1.6190 & 1 \end{bmatrix},\ K = \begin{bmatrix} 0 & 0 \\ 0 & 2.2212 \end{bmatrix},\ B_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix},\ \text{and}\ x = \begin{bmatrix} \theta \\ q \end{bmatrix}$$

where θ is the angle of the spacecraft hub in radians and q is a state variable representing appendage deformation using modal coordinates. From Equation (4.34), the state space dynamics are derived and are written as

$$\begin{cases} \dot{x}(t) = A_p x(t) + B_p u(t) \\ y(t) = C_p x(t) \end{cases}$$

where

$$A_p = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -2.166 & 0 & 0 \\ 0 & -2.5718 & 0 & 0 \end{bmatrix},$$

$$B_p = \begin{bmatrix} 0 \\ 0 \\ 0.06023 \\ 0.09751 \end{bmatrix},$$

and

$$C_p = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}$$

where $y(t) = \theta(t)$ is the measured angular position.

With appropriate substitutions, the transfer function of the plant becomes

$$y(t) = \frac{B(p)}{A(p)} u(t) = \frac{0.06023 p^2 + 0.13378}{p^4 + 2.57184 p^2} u(t) \tag{4.35}$$

It can be seen that, this flexible model has two non-minimum phase zeros located on the imaginary axis. The root locus plot of this plant is shown in Figure 4.14.

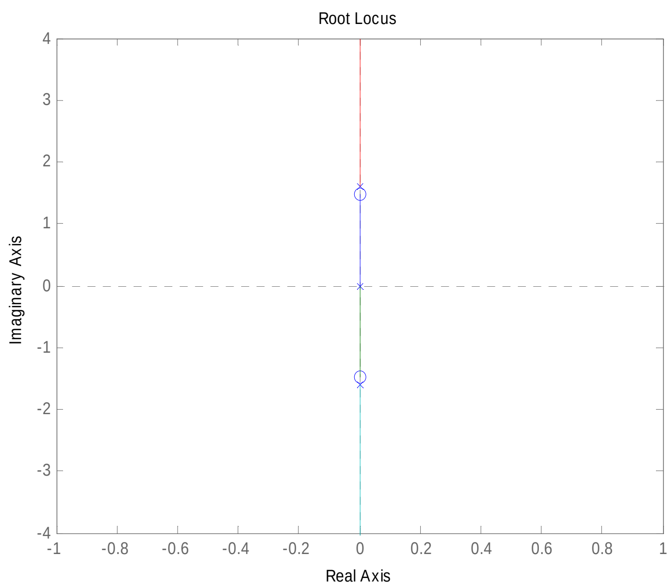

Figure 4.14. The Root Locus Plot of the Non-minimum System $y(t) = \dfrac{0.06023 p^2 + 0.13378}{p^4 + 2.57184 p^2} u(t)$.

It can also be verified that the PID controller with transfer function

$$\frac{D_1(p)}{E_1(p)} = \frac{10p^2 + 5p + 0.1}{p} \tag{4.36}$$

stabilizes the system. Also, it provides sufficient margin so that parameter perturbations do not affect the stability. Then, the nominal augmented system is

$$\bar{y}(t) = \left(k_0 \frac{B(p)}{A(p)} + \frac{E_1(p)}{D_1(p)} \right) u(t) \tag{4.37}$$

and becomes

$$\bar{y}(t) = \bar{k}_0 \frac{BB(p)}{AA(p)} u(t) = \frac{p^5 + 0.6023p^4 + 2.8730p^3 + 1.3438p^2 + 0.6689p + 0.0134}{p^6 + 0.5p^5 + 2.5818p^4 + 1.2859p^3 + 0.0257p^2} u(t) \tag{4.38}$$

where $k_0 = 1$ and $\bar{k}_0 = 0.1$.

Since the relative degree is $N - M = 1$, the reference model is chosen as

$$y_m(t) = \frac{1}{p+5} v(t) \tag{4.39}$$

with $v(t)$ representing an arbitrary external input. It can be easily verified that the augmented system is minimum phase, and can be adaptively controlled to follow the trajectory of the reference model of Equation (4.39).

Proceeding as in the previous chapter, we choose the filters $C_F(s)$ and $C_E(s)$ for the purposes of control and adaptation. With $\omega_0 = 30$ rad/sec, $\omega_1 = 30$ rad/sec, and $\omega_E = 50$ rad/sec chosen, the LPF becomes

$$C_F(p) = \left(\frac{\omega_0}{p + \omega_0} \right) \left(\frac{\omega_1^2}{(p + \omega_1)^2} \right) = \left(\frac{30}{p+30} \right) \left(\frac{30^2}{(p+30)^2} \right) \tag{4.40}$$

and

$$C_E(p) = \frac{{\omega_E}^L}{(p+\omega_E)^L} = \frac{50^2}{(p+50)^2}. \tag{4.41}$$

From the plot of $|H_{yy}(j\omega)|$ in Figure 4.15, it can be see that the magnitude is always smaller than "one", thus guaranteeing stability of the transfer functions : $H_{yy}(p)$, $H_{y0}(p)$, $H_{u0}(p)$, and $H_{uy}(p)$ in Equation (3.42).

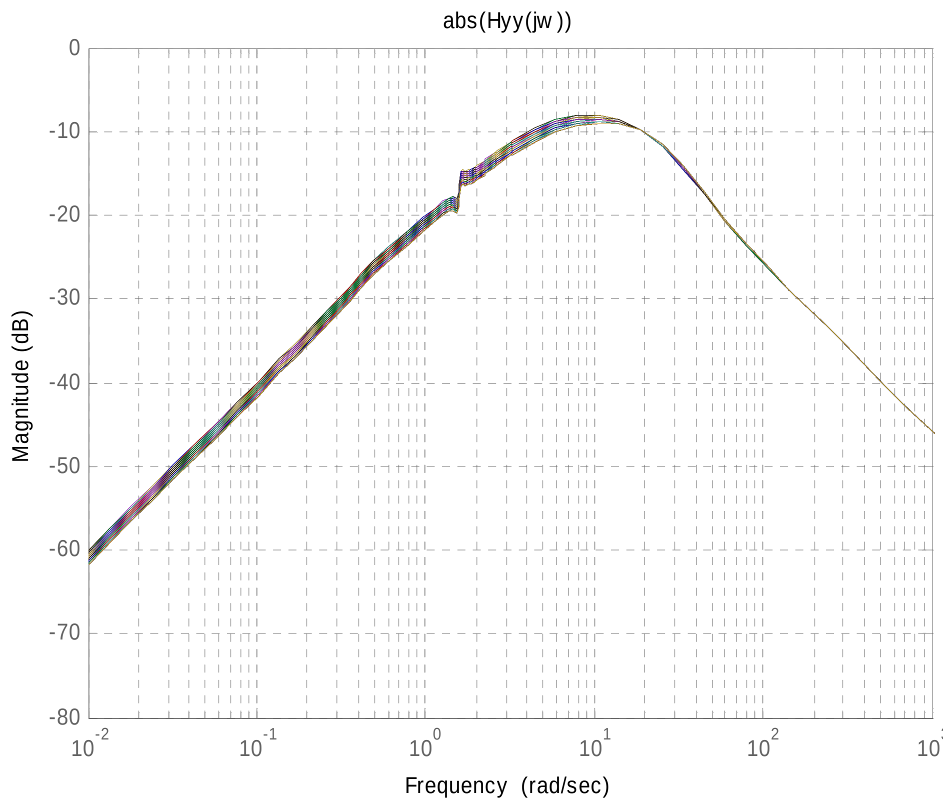

Figure 4.15. The Plot of the $|H_{yy}(j\omega)|$.

Figures 4.16 and 4.17 show the results of this experiment. Figure 4.16 shows the trajectories of the reference model, original model, and the modified plant. Figure 4.17 shows the results of control input, reference output, modified plant output, and original plant output, respectively. From these figures, it can be seen that the modified plant with PID control results in the plant becoming stable. Comparing the figure of the reference

output, it is evident that the modified output after adding the PID does track the trajectory of the reference model. In addition, no large control action or poor transient performance is exhibited during the entire execution period. This result validates that the modified adaptive controller with PID design does solve the flexible problems in non-minimum phase systems. This is the reason that such a controller is implemented in this experimental adaptive system.

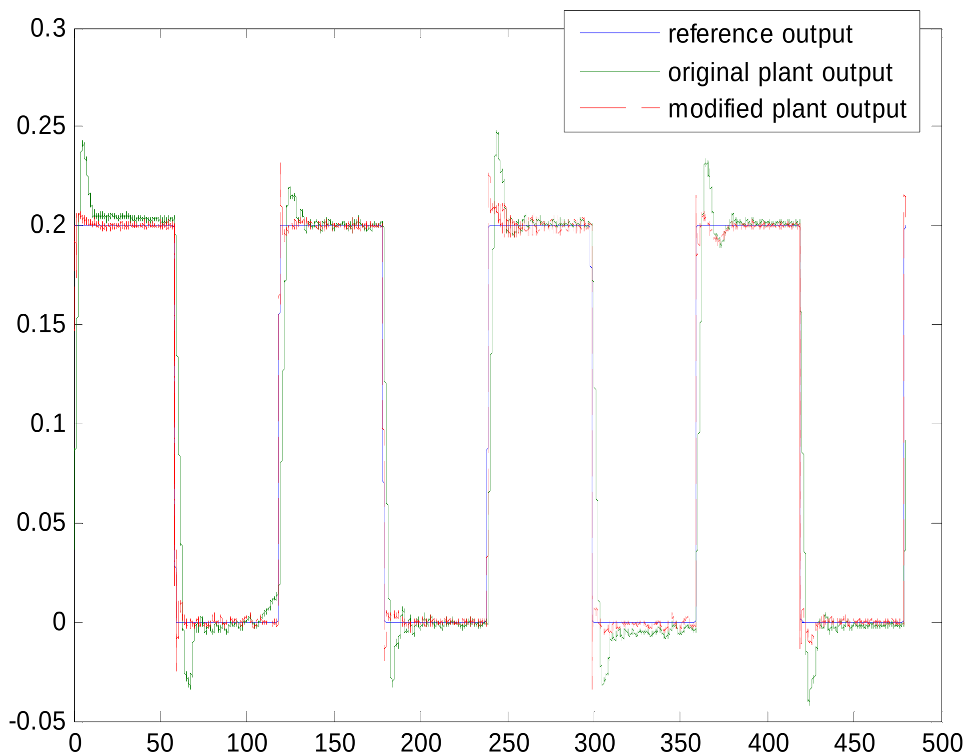

Figure 4.16. Trajectories of the Reference Model, Original Plant, and Modified Plant with Modified L1 Adaptive Controller and FSS.

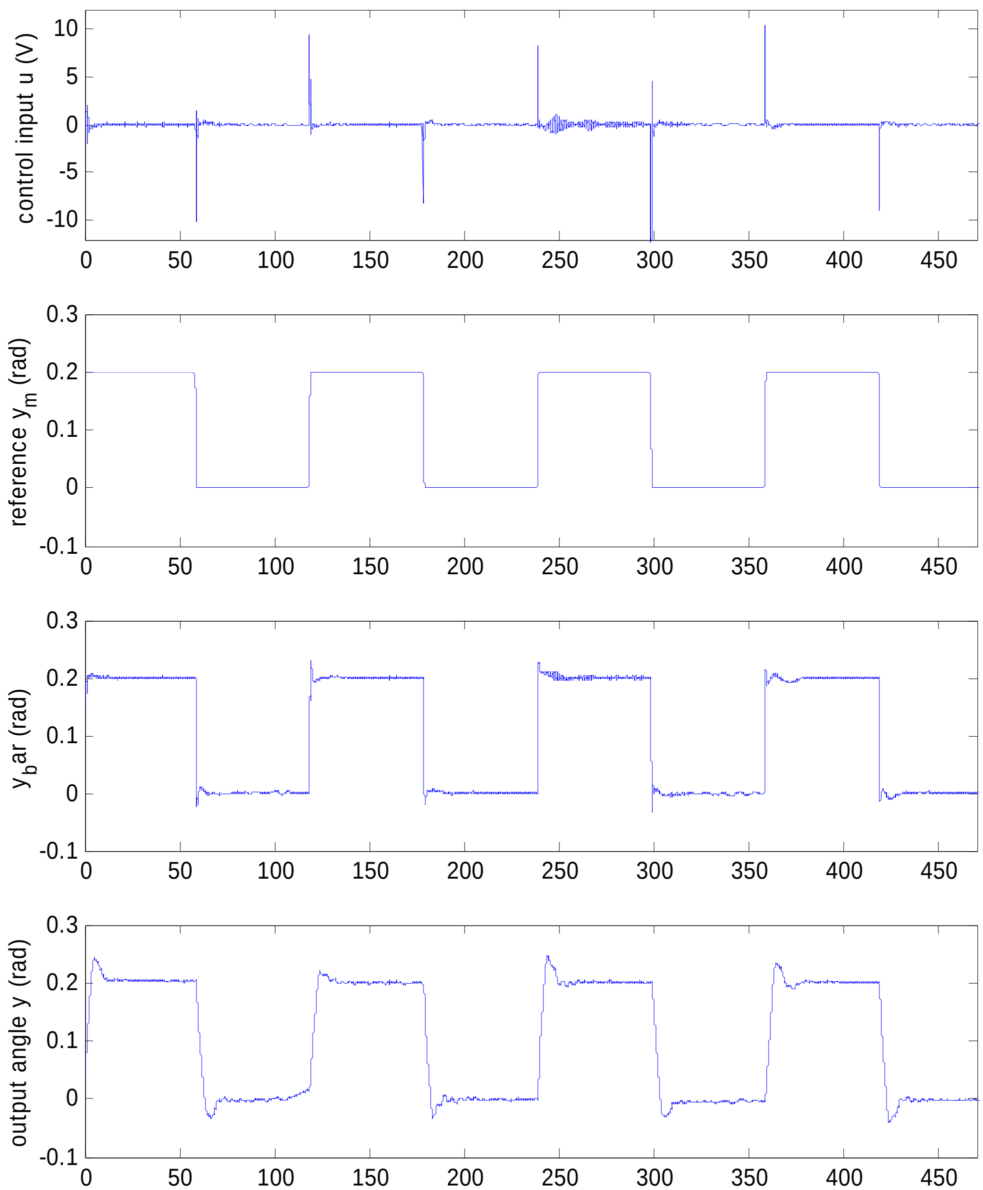

Figure 4.17. Experimental Results with Modified L1 Adaptive Controller and FSS.

V. CONCLUSIONS

A. SUMMARY AND CONTRIBUTIONS

In this dissertation, an Adaptive Control system has been redesigned for Single Input Single Output (SISO) systems to improve both transient response and robustness to external disturbances and unmodeled dynamics. In particular, it was shown how two properly designed filters, one in the control loop and one in the parameter estimation block, yield a transient response due to adaptation with an arbitrarily small maximum norm. Also, based on a number of simulations, it was verified that the system remained stable in the presence of external disturbances and unmodeled dynamics.

This approach was extended to non-minimum phase systems by assuming some stability conditions using a non-proper compensator, such as a PID controller. In the latter case, the effectiveness of this approach was demonstrated by application to a system with flexibilities, having a non-minimum phase transfer function.

The contributions in this dissertation are as follows:

1. For minimum phase, SISO systems global stability has been established together with criteria on the choice of the various filter parameters. Also, upper bounds of the transient response due to adaptation have been computed and shown how they are affected by the filters' bandwidths;
2. For non-minimum phase systems, we have shown that the use of an augmenting system in parallel with the plant makes the overall system minimum phase, so that the Adaptive Controller can be applied. The criterion of choosing the augmenting network is by the inverse of a PID controller, which stabilizes the plant. Although the PID controller has a non-proper transfer function due to the derivative action, its inverse is proper thus easily implementable in the presence of additive disturbances. The augmenting network is chosen so that it has no effect in steady state, only during the transient response of the system;
3. Application to a very lightly damped flexible system with two degrees of freedom on a plane shows the effectiveness of this approach in an actual implementation.

B. FURTHER WORK

The study in this dissertation presents a preliminary investigation on an Adaptive Control system with the intent of making it attractive to engineering problems. There are still a number of issues that need to be addressed, namely:

1. Robustness in the presence of external disturbances and unmodeled dynamics. Although simulation results show that this controller still keeps its stability properties with modeling errors, a need to establish how mush uncertainties can be tolerated while still guarantee stability has to be addressed. In particular the claim of arbitrarily small transient errors due to adaptation has to be reformulated to take modeling errors into account. The expected final result will be a usual tradeoff between performance and robustness;
2. The extension of this algorithm to non-minimum phase systems seems to be very important in view of the fact that systems with lightly damped flexibilities are very hard to control. The algorithm presented assumes that a stabilizing non-proper controller (such the PID) is known to the designer. The issue to be further investigated is the best way of determining this stabilizing controller, in view of the given uncertainties of the system to be controlled. From an engineering standpoint this would correspond to the combination of two controllers: a standard fixed PID control just to stabilize the system and the adaptive controller to improve its dynamic performance. These two controllers (fixed and adaptive) interact with each other, but their effects on the system performance and robustness need to be further investigated;
3. Finally most systems of interest have Multiple Inputs and Multiple Outputs (MIMO). This is the case of the flexible arm presented in the last section: extension to an actual implementation in space would require dynamics in all three dimensions with actuators in all three axes. For this sort of problems there are two possible ways of extending the theory and design. One way is to take the whole dynamics of the system into account and extend the SISO approach to MIMO. Along these lines a considerable amount of work has been done for standard Adaptive Control systems and it can be extended to the L1 approach. However in some cases as in the flexible arm in space, the MIMO system can be modeled as a set of SISO systems with cross coupling dynamics. In this case it might be easier to design separate independent SISO controllers and treat the cross couplings as disturbances. In this way, if the SISO controllers have sufficient degree of robustness the overall system will still be stable. These show important tradeoffs of this class of design.

APPENDIX A.

Equation (3.31) can be written as

$$\dot{\tilde{\theta}}_i(t) = -\mu \frac{X_i(t)}{\sigma^2 \left|\overline{w}_E(t)\right|^2 + \left\|X(t)\right\|^2} e(t) + \left(\mu - \mu_i(t)\right) \frac{X_i(t)}{\sigma^2 \left|\overline{w}_E(t)\right|^2 + \left\|X(t)\right\|^2} e(t) \qquad \text{(A.1)}$$

where $\mu - \mu_i(t) > 0$.

Define the positive define function

$$V(\tilde{\theta}(t)) = \frac{1}{2}\tilde{\theta}(t) \bullet \tilde{\theta}(t) \qquad \text{(A.2)}$$

then

$$\dot{V}(\tilde{\theta}(t)) = \tilde{\theta}^T(t) \bullet \dot{\tilde{\theta}}(t) \qquad \text{(A.3)}$$

and

$$\dot{V}(\tilde{\theta}(t)) = -\mu \frac{\tilde{\theta}^T(t) X(t)}{\sigma^2 \left|\overline{w}_E(t)\right|^2 + \left\|X(t)\right\|^2} e(t) + \sum_{i=0}^{2N} \left(\mu - \mu_i(t)\right) \frac{X_i(t)}{\sigma^2 \left|\overline{w}_E(t)\right|^2 + \left\|X(t)\right\|^2} e(t). \qquad \text{(A.4)}$$

The rightmost term is nonzero only when

$$\begin{cases} \tilde{\theta}_i(t) = \theta - \theta_{i,\max} \le 0 \ \text{ and } \ X_i(t)e(t) > 0 \\ \text{or} \\ \tilde{\theta}_i(t) = \theta - \theta_{i,\min} \ge 0 \ \text{ and } \ X_i(t)e(t) < 0 \end{cases} \qquad \text{(A.5)}$$

which makes it always non-positive.

Substituting for the error term $e(t) = \tilde{\theta}^T(t) X(t) + w_E(t)$ the following expression is obtained

$$\dot{V}(\tilde{\theta}(t)) \le -\mu\left(\varepsilon(t)^2 + \varepsilon(t)\eta(t)\right) = -\mu\left|\varepsilon(t) + \frac{1}{2}\eta(t)\right|^2 + \frac{1}{4}\left|\eta(t)\right|^2 \qquad \text{(A.6)}$$

where

$$\begin{cases} \varepsilon(t) = \dfrac{\tilde{\theta}^T(t)X(t)}{\sqrt{\sigma^2\left|\overline{w}_E(t)\right|^2 + \left\|X(t)\right\|^2}} \\ \\ \eta(t) = \dfrac{w_E(t)}{\sqrt{\sigma^2\left|\overline{w}_E(t)\right|^2 + \left\|X(t)\right\|^2}}.\varepsilon(t) = \dfrac{\tilde{\theta}^T(t)X(t)}{\sqrt{\sigma^2\left|\overline{w}_E(t)\right|^2 + \left\|X(t)\right\|^2}} \end{cases} \tag{A.7}$$

Now, with any t, T > 0 and integrating Equation (A.6) between t and t+T

$$0 \le V(\tilde{\theta}(t+T)) \le V(\tilde{\theta}(t)) - \mu \int_t^{t+T} \left|\varepsilon(\tau) + \frac{1}{2}\eta(\tau)\right|^2 d\tau + \frac{\mu}{4}\int_t^{t+T} \left|\eta(\tau)\right|^2 d\tau \tag{A.8}$$

yields

$$\int_t^{t+T} \left|\varepsilon(\tau) + \frac{1}{2}\eta(\tau)\right|^2 d\tau \le \frac{\left\|\tilde{\theta}(t)\right\|^2}{2\mu} + \frac{\mu}{4}\int_t^{t+T} \left|\eta(\tau)\right|^2 d\tau. \tag{A.9}$$

Definition of $\eta(t)$ in Equation (A.7) yields the upper bound on the rightmost term as

$$\int_t^{t+T} \left|\eta(\tau)\right|^2 d\tau \le \int_t^{t+T} \frac{\left|w_E(\tau)\right|^2}{\sigma^2\left|\overline{w}_E(\tau)\right|^2} d\tau \le \begin{cases} 0 \quad \text{if } w_E(\tau) = 0 \text{ for all } \tau \\ \\ \dfrac{T}{\sigma^2} \text{ otherwise.} \end{cases} \tag{A.10}$$

The final results become

a. Equation (3.33a), come easily since the initial estimates are within the bounds

$\theta_{i,\min} \le \hat{\theta}(t) \le \theta_{i,MAX}$, and

$$\begin{aligned} \hat{\theta}_i(t) \ge \theta_{i,MAX} &\Rightarrow \dot{\hat{\theta}}_i(t) \le 0 \\ \hat{\theta}_i(t) \le \theta_{i,m} &\Rightarrow \dot{\hat{\theta}}_i(t) \ge 0 \end{aligned} \tag{A.11}$$

b. From the definition of $\varepsilon(t)$ in Equation (A.7)

$$\left|\tilde{\theta}^T(t)X(t)\right| \le \left|\varepsilon(t)\right|\left(\sigma^2\left|\overline{w}_E(t)\right|^2 + \left\|X(t)\right\|^2\right)^{\frac{1}{2}} \tag{A.12}$$

where we can bound

$$\left(\int_t^{t+T}\left|\varepsilon(\tau)\right|^2 d\tau\right)^{\frac{1}{2}} \le \left(\int_t^{t+T}\left|\varepsilon(\tau)+\frac{1}{2}\eta(\tau)\right|^2 d\tau\right)^{\frac{1}{2}} + \left(\int_t^{t+T}\left|\frac{1}{2}\eta(\tau)\right|^2 d\tau\right)^{\frac{1}{2}} \tag{A.13}$$

Finally, combining Equations (A.9), (A.12), and (A.13) yields Equation (3.33b) with bound Equation (3.34).

c. Since

$$\left\|\dot{\tilde{\theta}}(t)\right\|^2 \le \mu^2 \frac{\left\|X(t)\right\|^2\left|e(t)\right|^2}{\left(\sigma^2\left|w_E(t)\right|^2 + \left\|X(t)\right\|^2\right)^2} \le \mu^2 \frac{\left|e(t)\right|^2}{\sigma^2\left|w_E(t)\right|^2 + \left\|X(t)\right\|^2} \tag{A.14}$$

and substituting for $e(t) = \tilde{\theta}^T(t)X(t) + w_E(t)$ yields

$$\frac{e(t)}{\sqrt{\sigma^2\left|\overline{w}_E(t)\right|^2 + \left\|X(t)\right\|^2}} = \varepsilon(t) + \eta(t) = \left(\varepsilon(t) + \frac{1}{2}\eta(t)\right) + \frac{1}{2}\eta(t) \tag{A.15}$$

This expression has the same upper bound as Equation (A.13) so that Equation (3.33c) follows easily with bound again of Equation (3.34)

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APPENDIX B.

Proof: From Equation (3.20), the expression for the plant output $y(t)$ is

$$y(t) = \frac{1}{D(p)}(k_m u(t) + (k_0 - k_m)u(t) + h^T \phi_u(t) + k^T \phi_y(t)) + w(t). \tag{B.1}$$

Combining Equations (3.21), (3.22), and (3.23), and rearranging Equation (3.20) as follows yields

$$\theta^T \phi(t) = D(p)\big(y(t) - w(t)\big) + (k_0 - k_m)u(t) - k_0 u(t). \tag{B.2}$$

From Equation (3.1)

$$k_0 u(t) = \frac{A(p)}{B(p)}\big(y(t) - w(t)\big). \tag{B.3}$$

Substituting Equation (B.3) into Equation (B.2) results in

$$\theta^T \phi(t) = D(p)y(t) - D(p)w(t) + (k_0 - k_m)u(t) - \frac{A(p)}{B(p)}y(t) + \frac{A(p)}{B(p)}w_0(t). \tag{B.4}$$

hen

$$\theta^T \phi(t) = D(p)\left(1 - \frac{A(p)}{B(p)D(p)}\right)y(t) + (k_0 - k_m)u(t) - D(p)w(t) + \frac{A(p)}{B(p)}w_0(t). \tag{B.5}$$

Substituting Equation (B.5) into Equation (B.2) yields

$$\begin{aligned} y(t) = {} & \frac{C_F(p)}{D(p)}(\tilde{\theta}^T(t)\phi(t) + v(t)) + \big(1 - C_F(p)\big)\left(1 - \frac{A(p)}{B(p)D(p)}\right)y(t) \\ & + \frac{\big(1 - C_F(p)\big)}{D(p)}(k_0 - k_m)u(t) - \big(1 - C_F(p)\big)w(t) + \frac{\big(1 - C_F(p)\big)}{D(p)}\frac{B(p)}{A(p)}w_0(t) + w(t) \end{aligned} \tag{B.6}$$

$$k_m u(t) = C_F(p)\left(\tilde{\theta}^T(t)\phi(t) + v(t)\right) - C_F(p)D(p)\left(1 - \frac{A(p)}{B(p)D(p)}\right)y(t)$$
$$-C_F(p)(k_0 - k_m)u(t) + C_F(p)D(p)w(t) - C_F(p)\frac{A(p)}{B(p)}w_0(t). \tag{B.7}$$

Equation (B.6) can be adjusted as

$$y(t) = \left(1 - C_F(p)\right)\left(1 - \frac{A(p)}{B(p)D(p)}\right)y(t) + \frac{\left(1 - C_F(p)\right)}{D(p)}\left(\gamma k_m u(t)\right) + \frac{C_F(p)}{D(p)}(\tilde{\theta}^T(t)\phi(t) + v(t))$$
$$-w(t) + C_F(p)w(t) + \left(1 - C_F(p)\right)\left(\frac{A(p)}{B(p)D(p)}\right)w_0(t) + w(t)$$

then

$$y(t) = \left(1 - C_F(p)\right)\left(1 - \frac{A(p)}{B(p)D(p)}\right)y(t) + \frac{\left(1 - C_F(p)\right)}{D(p)}\left(\gamma k_m u(t)\right) + \frac{C_F(p)}{D(p)}(\tilde{\theta}^T(t)\phi(t) + v(t))$$
$$+\left(1 - C_F(p)\right)\left(\frac{A(p)}{B(p)D(p)}\right)w_0(t) + C_F(p)w(t) \tag{B.8}$$

and finally

$$k_m\left(1 + \gamma C_F(p)\right)u(t) = -C_F(p)D(p)\left(1 - \frac{A(p)}{B(p)D(p)}\right)y(t) + C_F(p)\left(\tilde{\theta}^T(t)\phi(t) + v(t)\right)$$
$$-C_F(p)\left(\frac{A(p)}{B(p)}w_0(t) - D(p)w(t)\right). \tag{B.9}$$

Then $k_m u(t)$ is isolated:

$$k_m u(t) = \frac{-C_F(p)D(p)}{\left(1 + \gamma c_F(t)\right)}\left(1 - \frac{A(p)}{B(p)D(p)}\right)y(t) + \frac{C_F(p)}{\left(1 + \gamma c_F(t)\right)}\left(\tilde{\theta}^T(t)\phi(t) + v(t)\right)$$
$$-\frac{C_F(p)}{\left(1 + \gamma c_F(t)\right)}\left(\frac{A(p)}{B(p)}w_0(t) - D(p)w(t)\right). \tag{B.10}$$

Substituting $k_m u(t)$ from Equation (B.10) into Equation (B.8), results in the proof of Equations (3.41) and (3.42).

$$
\begin{aligned}
y(t) &= H_{yy}\left(p\right)y(t) + H_{yo}\left(p\right)C_F(p)\left(\tilde{\theta}^T(t)\phi(t) + v(t)\right) + H_{yw}\left(p\right)w_0(t) \\
k_m u(t) &= H_{uy}\left(p\right)y(t) + H_{uo}\left(p\right)C_F(p)\left(\tilde{\theta}^T(t)\phi(t) + v(t)\right) + H_{uw}\left(p\right)w_0(t)
\end{aligned}
\tag{3.41}
$$

where

$$
\begin{aligned}
H_{yy}\left(p\right) &= \left(1 - C_F\left(p\right)\right)\left(1 - \frac{A(s)}{B(p)D(p)}\right)\left(\frac{1}{1+\gamma C_F\left(p\right)}\right) \\
H_{y0}\left(p\right) &= \frac{\left(1+\gamma\right)}{\left(1+\gamma C_F\left(p\right)\right)D(p)} \\
H_{yw}\left(p\right) &= \frac{\left(1 - C_F\left(p\right)\right)}{\left(1+\gamma C_F\left(p\right)\right)}\left(\frac{A(p)}{B(p)D(p)} + \frac{k(p)}{P(p)D(p)} - 1\right) + 1 - \frac{k(p)}{P(p)D(p)} \\
H_{u0}\left(p\right) &= \frac{1}{1+\gamma C_F\left(p\right)} \\
H_{uy}\left(p\right) &= \frac{C_F\left(p\right)D(p)}{1+\gamma C_F\left(p\right)}\left(\frac{A(p)}{B(p)D(p)} - 1\right) \\
H_{uw}\left(p\right) &= \frac{C_F\left(p\right)D(p)}{1+\gamma C_F\left(p\right)}\left(\frac{A(p)}{B(p)D(p)} + \frac{k(p)}{P(p)D(p)} - 1\right)
\end{aligned}
\tag{3.42}
$$

and $\gamma = \dfrac{k_0}{k_m} - 1$ in the interval $0 \le \gamma \le \gamma_{MAX} = \dfrac{k_M}{k_m} - 1$.

QED

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APPENDIX C.

Let

a. $C_E(p)$ be the transfer function of an exponentially stable, causal Linear Time Invariant (LTI) system,

b. $\tilde{\theta}(t) \in R^N$ be a bounded vector

c. $\phi(t) \in R^N$, $\tilde{e}(t) \in R$ be the input and output of the following Linear Time Varying operator

$$\tilde{e}(t) = C_E(p)\left(\tilde{\theta}^T(t)\phi(t)\right) - \left(\tilde{\theta}^T(t)C_E(p)\phi(t)\right) \tag{C.1}$$

then

$$\tilde{e}(t) = \int_0^t c_E(t-\tau)\tilde{\theta}^T(\tau)\phi(\tau)d\tau - \int_0^t c_E(t-\tau)\tilde{\theta}^T(t)\phi(\tau)d\tau \tag{C.2}$$

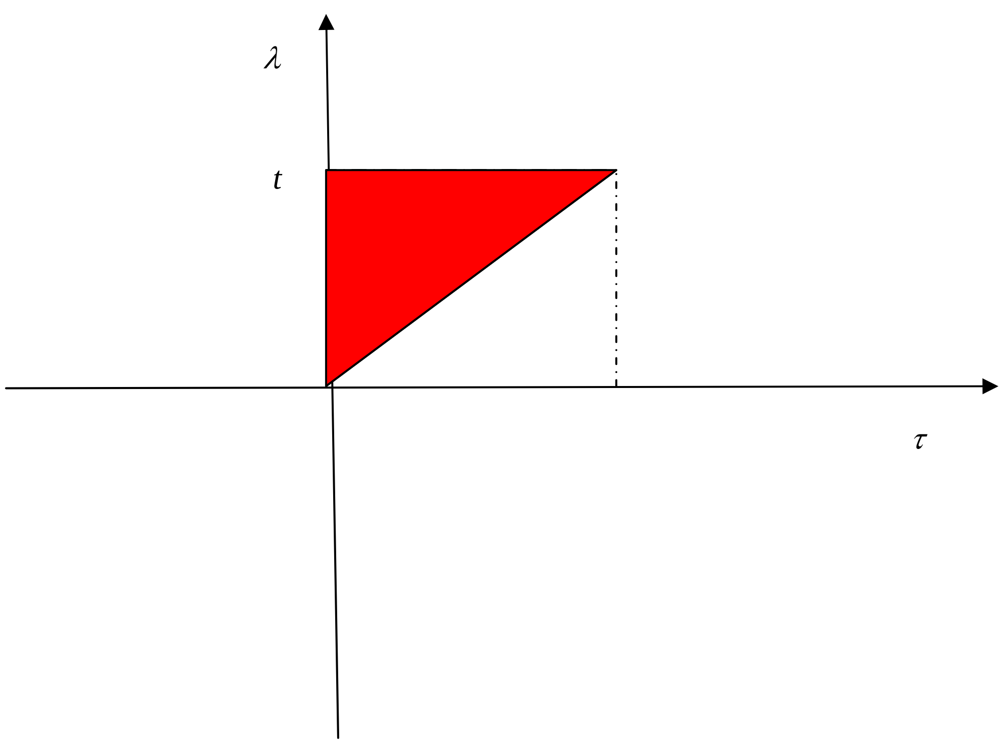

Figure C.1. Region of Integration for Equation (C.2).

Since

$$\tilde{\theta}(t) = \tilde{\theta}(0) + \int_0^t \dot{\tilde{\theta}}(\lambda) d\lambda \tag{C.3}$$

Substituting Equation (C.3) into Equation (C.2) yields

$$\tilde{e}(t) = -\int_0^t \int_\tau^t c_E(t-\tau)\dot{\tilde{\theta}}(\lambda)\phi(\tau) d\lambda d\tau \tag{C.4}$$

It is easy to see from Figure C.1 that the region of integration can be expressed as

$$\begin{aligned} 0 \le \lambda \le t \\ 0 \le \tau \le \lambda \end{aligned} \tag{C.5}$$

which yields

$$\tilde{e}(t) = -\int_0^t \dot{\tilde{\theta}}^T(\lambda) \int_0^\lambda c_E(t-\tau)\phi(\tau) d\tau d\lambda \tag{C.6}$$

A simple change of integration variables yields the result.

QED

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